Fall 2013

18.310 Homework 12

Due Wednesday December 4th at 6PM

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1. Suppose you are encoding a source that emits one of three letters: a with probability $\frac{1}{2}$, b with probability $\frac{1}{3}$ and c with probability $\frac{1}{6}$.

   (a) What is the Shannon bound on the best encoding of n letters from this source.

   (b) Use the Huffman algorithm to find an optimal prefix code for encoding this source. What is the number of bits used per letter?

2. Now, consider the same source as in problem (1), but the new 9-letter "alphabet" consisting of all pairs of letters, so aa would have probability $\frac{1}{4}$, ab would have probability $\frac{1}{6}$, etc.

   (a) What is the Shannon bound on the best encoding of n "letters" from this source.

   (b) Use the Huffman algorithm to find an optimal prefix code for encoding this source. What is the number of bits used per "letter'? Per letter of the original source?

3. Level-Ziv encoding will be covered on Monday.

   (a) Suppose you encode n digits from the sequence

   $$12345678910111213141516171819202122\cdots$$

   obtained by concatenating all natural numbers. Approximately how many bits will this take to encode using Lempel-Ziv? By approximately, we mean that we care only about the asymptotic growth as n gets large.

   (b) Suppose you encode n bits from the sequence

   $$01010101010101010101\cdots$$

   obtained by alternating 0's and 1's. Approximately how many bits will this take to encode using Lempel-Ziv?

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