Recap of points needed for problem sets.
Domain of dependence for $a(x,y)*u_x + b(x, y)*u_y = c(x, y)*u + d(x, y)$ with u given along some curve Gamma.

Region where the solution is defined by the characteristics that go through Gamma.
Example: $x*u_x + y*u_y = u$, with $u=1/(1+x^2)$ on $y = 1$.
Sln. determined for $y > 0$ only, even though the formula for u that one gets by solving the equation, $u = y^3/(x^2+y^2)$, has a value for any x, y, not both zero.

Point out: can "extend" this solution to the lower half plane $y < 0$ in many ways.
Example: $u = C*y^3/(x^2+y^2)$, C any constant, solves equation for $y < 0$, and matches above solution with continuous derivatives!

Note, general solution is $u = r*f(\theta)$ in polar coordinates, because equation is $r*u_r = u$.
Any f that vanishes and has a derivative that vanishes, at $\theta = 0$ and pi, can be used to extend solution below!

Another point: A student asked during lecture about shocks and shock crossings:
What if the characteristics hit each other "head on"?
An alternative way to put the question is:
Imagine a characteristic that "turns around in time", what do you do?
Example: a curve like $t = t0 - x^2$, $-\sqrt{t0} <= x <= \sqrt{t0}$.
Time must proceed forward, so this is 2 characteristics.
1) $x = + \sqrt{t0-t}$, for $0 <= t <= t0$ [x decreasing!]
2) $x = - \sqrt{t0-t}$, for $0 <= t <= t0$ [x increasing!]

At $t = t0$ these two characteristics collide, head on, and kill each other.

- --- This provided that a shock did not cut them off earlier.
- --- At $x = 0$, the solution will, generally, have some singular behavior, as it is getting info from two different characteristics.

An example of this type occurs in the problem
Linear 1st order PDE # 09 (surface evolution).
There the ICs are special, so no singularity occurs.
Then, back to shocks:
Shocks in the "green light turns red" traffic flow examples we did have the discontinuity "backwards". This is consistent with steepening. Shocks only needed in this case. Forward discontinuities self-destruct [red light turns green example].

What do you expect for river flows? Does it match observations?