a) Example 1:
Compare behavior, with initial smooth "bump" profile $\rho = f(x)$, or
$A = f(x)$ for TRAFFIC FLOW and FLOOD WAVES.
List resulting differences in behavior.

b) POINT OUT: method works for linear/semilinear/quasilinear scalar
eqns. Define linear/semilinear/quasilinear.

c) Characteristics almost always cross.
WRITE THE PRECISE CONDITIONS NEEDED FOR THIS TO HAPPEN.
WHEN/WHERE DO CHARACTERISTICS CROSS. FIRST CROSSING.

Simple problem: $\rho_t + q_x = \rho_t + c(\rho)*\rho_x = 0$,
$\rho(x, 0) = R(x)$,
$c(\rho) = dq/d\rho$.

Solution by characteristics: $x = X(s, t) = C(s)*t + s$, (#1)
$\rho = R(s)$,

where $C(x) = c(R(x))$ = wave speed along initial data.

Characteristics do not cross if and only if can solve for s as a function of x and t --- $s = S(x, t)$ --- from (#1) if and only if map s --> x is monotone: X_s not 0.

That is: inspect $X_s = C'(s)*t + 1$.

So, if $C'(s) < 0$ somewhere, there will be a time when $X_s = 0$.

Thus, the condition for crossing is:
$dc/dx < 0$ somewhere in the initial data.

Graphics: show how $x = X(s, t) = C(s)*t+s$ looks like as a function of s, for t fixed, as t grows, if C(s) is a localized hump.
t = 0: straight line x = s.
t > 0, moderate: straight line develops a wiggle.
t > 0, large: wiggle large enough to produce a local max. and a local min. Hence a range where map is not 1-to-1.

Formula for critical time t_c and location x_c where the characteristics cross first, assuming $C'(s) < 0$ somewhere (no crossings otherwise):
Let s_c be the value of s at which C'(s) reaches its largest negative value (i.e.: absolute minimum). Then $t_c = - 1/C'(s_c)$ ... so X_s vanishes.
$x_c = C(s_c)*t_c + s_c$.

Extras: To (graphically) visualize infinities of ρ_x and ρ_t, characterized by the solutions to $1+t*C'(s)$, plot $y = C'(s)$ versus $y = - 1/t$ (horizontal line).

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