18.315
9/7/05

won't talk much about CS app'ns, B's X book good for that

Prerequisite: must be mature + smart people, Know standard stuff: Cayley f'la$^{n^{n-2}}$ (spanning trees in K_n), Catalan #s$^{\frac{1}{n+1}\binom{2n}{n}}$, Ramsey Θ, generating f'ns, Euler's thm on Eulerian paths, graph of every convex polytope in $\mathbb{R}^3$ is 3-conn., Chebychev + Markov ineq., groups...

Borsuk Θm

$X \subseteq \mathbb{R}^2$ convex $\Rightarrow \exists\, X_1, X_2, X_3$ s.t. $X = X_1 \amalg X_2 \amalg X_3$

$\text{diam}(X_i) < \text{diam}(X)$ $i = 1,2,3$

In $\mathbb{R}^d$, it's Borsuk Conjecture ($d=3$ ✓, $d \gtrsim 300$ not true)

Dehn's Θm

Cor Every convex polytope in $\mathbb{R}^3$ is rigid (i.e. ~~can't continuously~~ if $P(t)$ is a family of polytopes s.t. ours $P(0) \in P(t)$ + all faces are isometric polygons, then only has rotations + reflections) (actually due to Cauchy)

Dehn's Thm (1915)
Every convex polytope in $\mathbb{R}^3$ is infinit. rigid

Ramsey's Thm! Yay!
$\forall r, k \quad \exists n = n(r,k)$ s.t. $\forall$ r-edge-coloring of K_n
$\exists$ mono K_k
Pf reminder: blah...

Extension:
$\forall r, k, s \quad \exists R_r(k,s)$ s.t. $\forall$ r-coloring of
k-subsets of $[n]$ $\exists$ s-subset in $[n]$ s.t. all
$\binom{s}{k}$ subsets are mono $(n \geq R_r(k,s))$
Proof: same

Schur's Thm
$\forall r \; \exists n = n(r)$ s.t. $\forall$ coloring of $[n]$ $\exists$ triple
$\{x, y, x+y\}$ monoch
Hack proof: Let $n(r) = R_r(2,3)$
$\chi : [n] \to [r]$ coloring, $\eta : [n]^2 \to [r]$
$\eta(x,y) = \chi(|x-y|)$ ✓