Pak
10/3/05

Tutte polynomial (cont'd)

$$T_G(x,y) = \sum_{H \subseteq G} (x-1)^{c(H)-c(G)} (y-1)^{|F|-|V|+c(H)}$$

$H = (F, V)$ $\quad F \subseteq E$ $\quad c(G) = \#$ conn. comp. of G

$$T_G = \begin{cases} x\,T_{G-e}(x,y) & e \text{ a bridge} \\ y\,T_{G-e}(x,y) & e \text{ a loop} \\ T_{G-e}(x,y) + T_{G/e}(x,y) & e \text{ neither} \end{cases}$$

Note: $T_G(2,2) = 2^m$ $\quad m = |E|$

$T_G(1,1) = \#$ spanning trees (G conn) (always)

Theorem: ~~Conjecture~~: $\exists\, \alpha, \beta : t \to \mathbb{Z}_+$ (sp. trees in G)

s.t. $T_G(x,y) = \sum_t x^{\alpha(t)} y^{\beta(t)}$

Let $<$ ~~linear~~ linear order G. $\quad \alpha = \#$ internally active $e \in t$

$\beta = \#$ externally " $e \in G-t$

e is externally active if it's the largest in a cycle obtained by adding e to t

Similarly, e is internally active if e is the largest in a $t-e$ cut. I.e., remove $e \Rightarrow$ 2 connected comp'ts in $t \Rightarrow$ can look at edges (in G) between them, want e to be bigg[est]

Proof: remove smallest edge, induct
(for bridge, loop, o/w) ✓

→ By next time, we ought to calculate $T_{C_n}(x,y)$

$$\left(T_{C_n} = x^{n-1} + T_{C_{n-1}}(x,y) \right)$$

Also $\chi_G(t) = (-1)^{|V|-c(G)} t^{c(G)}\, T_G(1-t, 0)$

(see Bollobás)