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Thm (non-uniform R-W)
p prime, $L \subseteq \mathbb{N}$, $|L| = s$, $\mathcal{F} = \{A_1, \dots A_m\}$
$\forall i,j$ $A_i \subseteq [n]$, $|A_i| \notin L \mod p$, $|A_i \cap A_j| \in L \mod p$
Then $m \le \binom{n}{s} + \binom{n}{s-1} + \dots + \binom{n}{1} + \binom{n}{0} \left(\le \binom{n}{s}\left(1 + \frac{s}{n-2s+1}\right)\right)$

Tool: multilinearization criteria
$\mathbb{K}$ fixed field, $\Omega = \{0,1\}^n \subset \mathbb{K}^n$, f poly'l of deg $\le s$ w/ n variables. Then $\exists (!)$ poly'l g s.t. deg $\le s$, n var, $f(v) = g(v)$ $\forall v \in \Omega$, g is multilinear (i.e. linear in each variable)
Pf: Ignore powers (since they're irrelevant on Ω) ✓

Pf: $v_i = \chi(A_i) \in \mathbb{F}_p^n$, $F(x,y) = \prod_{l \in L} (x \cdot y - l)$
where $x = x_1 \dots x_n$, $x \cdot y = x_1 y_1 + \dots + x_n y_n$
$f_i(x) = F(x, v_i)$ Note $f_i(v_j) = \begin{cases} 0 & i \ne j \\ \ne 0 & \text{o/w} \end{cases}$
$\Rightarrow \{f_i\}$ lin indep (as before).
Now replace f_i w/ g_i, which are lin indep for the same reason, dim (spanned by $\{g_i\}$) $\le \binom{n}{s} + \binom{n}{s-1} + \dots + \binom{n}{0}$
since basis $= \{1\} \cup \{x_i\} \cup \{x_i x_j\} \cup \dots \cup \{x_{i_1} \dots x_{i_s}\}$ ✓

Chromatic # of $\mathbb{R}^n$
$V(\Gamma_n) = \mathbb{R}^n$ $E(\Gamma_n) = (v,v') \in \mathbb{R}^{2n}$ s.t. $d(v,v') = 1$
$\chi(\Gamma_n) = ??\dots?$

Theorem (FW)
$\chi(\Gamma_n) > (1+\varepsilon)^n$ $\varepsilon > 0$ some fixed constant

Cor: $\mathcal{F} = (2p-1)$-uniform family $\{A_1, \dots A_m\}$, $A_i \subseteq [4p-1]$,
$|A_i \cap A_j| \neq p-1 \Rightarrow |\mathcal{F}| = m \leq 2\binom{4p-1}{p-1} < 1.8^n$

Pf: $L = \{0, 1, \dots p-2\}$, $s = p-1$, $\Rightarrow m \leq \binom{n}{s}\left(1 + \frac{s}{n-2s+1}\right) \leq$
$2\binom{4p-1}{p-1}$

So look at vectors in $\{0,1\}^{4p-1}$ w/ $2p-1$ 1's in them
Have a bound on how many have dist
$\neq \sqrt{4p-2-(p-1)}$, since these correspond to $\mathcal{F}$ satisfying above, if we rescale to get $\sqrt{4p-2-(p-1)}$ to be unit distance, then we've got bound on largest indep. set, then can use $\alpha(G)$ bd on $\chi(G)$ ✓