Pak
10/28/05

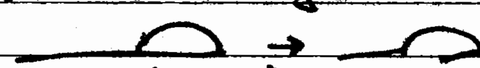
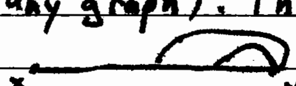
Thm (Tutte)
Every 4-connected planar graph is Hamiltonian (cycle-wise)

(might want to compare this w/ Whitney thm (ex. 4))

Thm ~~[illegible]~~
G a graph w/ all vertices of odd degree $\Rightarrow$ every edge in G belongs to an even # Hamiltonian cycles

Cor. ~~[illegible]~~ G cubic $\Rightarrow$ G has 0 or ≥ 3 H c (grrr...)

Thm $\Rightarrow$ Cor
Take an edge e in a h.c. in G. $\exists$ another cycle w/ e in it, $\exists$ e' that's in one but not the other, do the same thing again ✓

Pf of thm: Let $\mathcal{P} = \{$longest paths starting at $x \in V(G)\}$
Let H be a graph $V(H) = \mathcal{P}$ & edges correspond to simple transformations 
Claim: $W = \{$vertices of G w/ even degree$\}$ (we're letting G be any graph). Then # $p \in \mathcal{P}$ ending at W is even
Pf:  $y \in W$ all y's neighbors are w/in long path $\Rightarrow$ deg y − 1 transforms
$\Rightarrow$ deg of that path in H is odd
And if deg y were ~~[illegible]~~ odd then deg in H would be even ✓

~~Defn~~ Let $G' = G - e$ (where G is our odd degree graph), let $e = \{xy\}$, in pf let x be starting vx, now $W = \{y\}$, so # longest paths $x \to y$ in G' is even. If G' is hamiltonian, then longest paths are h'n, so e is in an even # of h.c. ✓

Unrealized homework:

1) $\Gamma(S_n, \{(12), (23), \ldots (n-1, n)\}) = \Gamma_n$ i.e.

$V = S_n \quad E = \{(\sigma, (i, i+1)\sigma)\}$

Thm Γ_n contains h.c. for $n \geq 3$

($\exists$ lots of proofs online, etc.)

2) $\Gamma_n = \Gamma(S_n, \{(12), (12\ldots n)^{\pm 1}\})$

Thm Γ_n contains HC

(too hard, Don Knuth has sol'n)

3) $\Gamma_n = \Gamma(S_{2n}, \{(12), (12)(34)\ldots(2n-1, 2n), (23)(45)\ldots(2n-3, 2n-2)\})$

Thm Γ_n contains HC

Beginning of the proof: G a finite group,

$G = \langle \alpha, \beta, \gamma \rangle$ and $\alpha^2 = \beta^2 = \gamma^2 = 1 = \alpha\beta\alpha^{-1}\beta^{-1}$

$\Gamma = \Gamma(G, \{\alpha, \beta, \gamma\})$ Cayley graph

$V(\Gamma) = \{g \mid g \in G\} \quad E(\Gamma) = \{(g, g\alpha), (g, g\beta), (g, g\gamma) \mid g \in G\}$

$\Rightarrow$ Γ contains H.C.

Note that Γ_n satisfies this, so it's enough

Pf: Next time