Pak

12/?/05

Last time's proof for TPT showed # integer pts is the same by constructing a map X_λ

X_λ is
0) maps integer pts to integer pts
1) volume preserving
2) piecewise linear
3) continuous

0) $\Rightarrow$ 1st (integer) part of thm,
1) $\Rightarrow$ 2nd part
1) is true because it's the composition of volume preserving maps (a bunch of reflections, in fact)

Thm $\forall$ $P, Q \subset \mathbb{R}^d$ convex polytopes if $vol(P) = vol(Q)$ then $\exists$ $\varphi: P \to Q$ satisfying 1, 2, 3

(Example I didn't write down, uh-oh, involving $J = (\ldots 1\ 2 \ldots n-1\ n\ n-1 \ldots 1 \ldots)$, and Gelfand Tseitlin patterns)

Corollary $n! = \sum_{\substack{\lambda \vdash n \\ (\text{partition of } n)}} |SYT(\lambda)|^2 = |S_n|$

$\forall \lambda \vdash n$ π_λ irr. rep. of S_n

$\dim(\pi_\lambda) = |SYT(\lambda)|$

Young

Prop'n: $\forall \lambda$, $s = (i,j)$ corner of λ (i.e. square w/ nothing right or below)

$\chi_\lambda^{-1}(B) = A \Rightarrow A(i,j) =$ sum along max'l path $(1,1) \to (i,j)$ in B (only right + down moves allowed)

Pf: By induction. Remember from last time
$B(i,j) = A(i,j) - \max\{A(i-1,j), A(i,j-1)\}$
and max path has to go through $(i-1,j)$ or $(i,j-1)$

(still true for s not a corner, but you have to be more careful) (in fact, he totally cops out and assumes λ is a square)

Example: $\lambda = (n \ldots n)$ $\quad J = (1\,2 \ldots n\; n-1 \ldots 1)$
$B =$ perm. matrix. then max path sum to (n,n) is max length of increasing subsequence

Corollary # $\sigma \in S_n$ w/ longest inc. subseq. of length ℓ

$$= \sum_{\substack{\lambda \vdash n \\ \lambda_1 = \ell}} |SYT(\lambda)|^2$$

Why? correspondence between squares + tableaux w/ max square $= \ell$
(same as RSK correspondence)