

18.335 Problem Set 2

Problem 1: Stability

- (a) Trefethen, exercise 15.1. [In parts (e) and (f), assume that $\frac{1}{k!}$ can be computed to $O(\epsilon_{\text{machine}})$ and concentrate on the accumulation of errors in the summations.]
- (b) Trefethen, exercise 16.1.

Problem 2: Norms

- (a) Derive Trefethen eq. (3.10) (for which Trefethen only writes “by much the same argument”). Find the code that computes the induced $\|A\|_{\infty}$ norm in Julia, the `opnorm(A, Inf)` function, on github.com/JuliaLang/julia in `stdlib/LinearAlgebra/src/generic.jl` and satisfy yourself that it is equivalent to (3.10).
- (b) Trefethen, problem 3.4. Check your result for a random 10×7 matrix A in Julia, constructed by `A=randn(10,7)` with the induced $p = 2$ norm as computed by `opnorm(A)` in Julia.

Problem 3: Conditioning

In class, we found the condition number of $f(x) = Ax$, in the L_2 norm, to be $\|A\|_2 \|x\|_2 / \|Ax\|_2$ for some $A \in \mathbb{C}^{m \times n}$ and $x \in \mathbb{C}^n$. Instead, compute the condition number of $g(A) = Ax$, using any convenient choice of norm (e.g. the Frobenius norm for the input matrices A).

Problem 4: SVD and low-rank approximations

- (a) Trefethen, problem 4.5.
- (b) Trefethen, problem 5.2.
- (c) Trefethen, problem 5.4.

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