

18.441. Pset 5.

(1) Find sufficient statistic for gamma distribution $\Gamma(\alpha, \beta)$, where the value of β is known and the value of α is unknown.

(2) Consider a family of distributions with p.d.f. given by

$$f(x|\theta) = a(\theta)b(x)e^{c(\theta)d(x)}$$

- the so called exponential family of distributions. Given an i.i.d. sample X_1, \dots, X_n with distribution from this family, find sufficient statistic.

(3) Find jointly sufficient statistics for gamma distribution $\Gamma(\alpha, \beta)$, where now both β and α are unknown.

(4) Suppose that we have an i.i.d. sample X_1, \dots, X_n from exponential distribution $E(\alpha)$ with α unknown. Is the MLE of α a minimal sufficient statistic?

(5) Consider an i.i.d. sample X_1, \dots, X_n from uniform distribution $U[0, \theta]$ and let

$$Y_n = \max(X_1, \dots, X_n).$$

Find the estimate of unknown θ of the form cY_n that minimizes the squared error loss

$$E_\theta(cY_n - \theta)^2.$$