

18.465 PS2, due Thursday, February 24, 2005

1. RAP, Sec. 11.9, p. 432, no. 3. The background of the problem (for motivation) is: in the first few decades of the twentieth century it was considered a very good property of an estimator to be unbiased. More recently, it was realized that unbiasedness sometimes forces a bad choice of estimator. (Compare also problem 4 on p. 432.)

In an experiment, some samples were radioactive, others not. All radioactive samples were assumed to have the same amount of a given radioisotope in them. The number of radioactive decay events observed from radioactive samples was assumed to have a Poisson distribution. But if no decays were observed, one wouldn't know if there was any radioisotope in the given sample. The experimenters wanted an estimate of the probability that a sample had radioisotope in it but was overlooked because it gave no decay events, given the numbers of decay events from other samples only when they were non-zero. So someone looked for an unbiased estimate, leading to the problem.

Now, in problem 3 as stated, on the left of the equation is a conditional expectation given that $Y \geq 1$, and $P(Y \geq 1) = 1 - P(Y = 0) = 1 - e^{-\lambda}$. So for $k = 1, 2, \dots$,

$$P(Y = k|Y \geq 1) = P(Y = k)/(1 - e^{-\lambda}) = e^{-\lambda} \lambda^k / (k!(1 - e^{-\lambda})),$$

so $E(V|Y \geq 1)$ has the given expression.

Hint: if two functions of λ given by power series (Taylor series) are equal (for all $\lambda > 0$), the coefficients must be equal.

The remaining four problems are from Randles and Wolfe.

2. Problem 2.4.10, p. 58 (table of data is on p. 59). A table for the signed rank test will be distributed.

3. Problem 3.1.9 p. 69. Further hint: first consider $U[0, b]$, evaluate $h(b)$ for all $b > 0$ by the fundamental theorem of calculus, then consider $U[a, b]$ for $0 < a < b$ to get a contradiction.

4. Problem 3.2.10 p. 77.

5. Problem 3.2.12 p. 77. *Hint:* Divide the numerator and denominator by \sqrt{n} . Apply one probability limit theorem in the numerator and a different one in the denominator. Use Slutsky's theorem 3.2.8 (which part?).