

## 18.703 HOMEWORK #2, DUE THURSDAY FEBRUARY 28TH

1. Let

$$Z(G) = \{ g \in G \mid hg = gh \quad \forall h \in G \}.$$

$Z(G)$  is called the **centre** of  $G$ .

(i) Show that

$$Z(G) = \bigcap_{g \in G} C_g.$$

(ii) Show that the centre  $Z(G)$  of  $G$  is a subgroup of  $G$ .

2. Herstein, Chapter 2, §4, 1, (b).

3. If  $G$  has no proper subgroups then show that  $G$  is cyclic of order  $p$ , where  $p$  is a prime number.

4. Herstein, Chapter 2, §4, 13.

5. Herstein, Chapter 2, §4, 14.

6. Find all subgroups of  $D_4$ , the group of symmetries of the square.

7. Herstein, Chapter 2, §4, 16.

8. Herstein, Chapter 2, §4, 24.

9. Herstein, Chapter 2, §4, 26.

10. Herstein, Chapter 2, §4, 27.

11. Herstein, Chapter 2, §4, 36.

12. Herstein, Chapter 2, §4, 37.

13. Herstein, Chapter 2, §4, 38.

14. **Challenge Problem:** Judson, Chapter 3, Question 50.

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