SECOND PRACTICE MIDTERM
MATH 18.703, MIT, SPRING 13

You have 80 minutes. This test is closed book, closed notes, no calculators.

There are 6 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.* Points will be awarded on the basis of neatness, the use of complete sentences and the correct presentation of a logical argument.

Name:

Signature:

Student ID #:

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	20	
6	15	
Presentation	5	
Total	100	

1. (15pts) (i) Give the definition of an irreducible element of an integral domain.

(ii) Give the definition of a prime element of an integral domain.

(iii) Give the definition of a principal ideal domain.

2. (15pts) (i) State the Sylow Theorems.

(ii) Prove that there is no simple group of order 120.

3. (15pts) Let R be an integral domain and let I be an ideal. Show that R/I is a field iff I is a maximal ideal.

4. (15pts) Let R be a ring.
(i) If

$$I_1 \subset I_2 \subset I_3 \subset \cdots \subset I_n \subset \cdots ,$$

is an ascending sequence of ideals then the union I is an ideal.

(ii) Show that if R is a PID then every ascending chain condition of ideals eventually stabilises.

5. (20pts) Let R be a ring and let I and J be two ideals.
(i) Show that the intersection $I \cap J$ is an ideal.

6 (ii) Is the union $I \cup J$ an ideal?

6. (15pts) Let R be a principal ideal domain and let a and b be two non-zero elements of R. Show that the gcd d of a and b exists and prove that there are elements r and s of R such that

$$d = ra + sb.$$