HOMEWORK 1 FOR 18.725, FALL 2015
DUE TUESDAY, SEPTEMBER 15 BY 1PM.

(1) Describe the sets of maximal ideals in the rings $\mathbb{R}[x]$, $\mathbb{F}_q[x]$.
Hint: The answer is a quotient of $\mathbb{C}$, respectively, $\bar{\mathbb{F}}_q$ by an equivalence relation.

(2) Let $k = \bar{\mathbb{Q}}$ and $R = k[x_1, \ldots, x_n, \ldots]$ be the ring of polynomials in infinitely many variables. For $\mathbf{a} = (a_i) \in \prod_{i=1}^{\infty} k$ we have a homomorphism $R \to k$ sending x_i to a_i, let $\mathfrak{m}_{\mathbf{a}}$ be its kernel. Find an example of a maximal ideal in R which is not of the form $\mathfrak{m}_{\mathbf{a}}$ for any $\mathbf{a} \in \prod_{i=1}^{\infty} k$.

(3) Show that $k[\mathbb{A}^2 \setminus \{0\}] = k[\mathbb{A}^2]$. Conclude that $\mathbb{A}^2 \setminus \{0\}$ is not affine.
[Hint: Use the covering by two affine open subsets given by $x \neq 0$ and $y \neq 0$, where x, y are coordinates on $\mathbb{A}^2$].

18.725 Algebraic Geometry
Fall 2015