Open Problem 2.1 *Recall the definition of $R(r)$ above, the following questions are open:*

- *What is the value of $R(5)$?*
- *What are the asymptotics of $R(s)$? In particular, improve on the base of the exponent on either the lower bound ($\sqrt{2}$) or the upper bound (4).*
- *Construct a family of graphs $G = (V, E)$ with increasing number of vertices for which there exists $\varepsilon > 0$ such that*[9]

$$|V| \lesssim (1+\varepsilon)^r.$$

It is known that $43 \leq R(5) \leq 49$. There is a famous quote in Joel Spencer's book [Spe94] that conveys the difficulty of computing Ramsey numbers:

"*Erdős asks us to imagine an alien force, vastly more powerful than us, landing on Earth and demanding the value of $R(5)$ or they will destroy our planet. In that case, he claims, we should marshal all our computers and all our mathematicians and attempt to find the value. But suppose, instead, that they ask for $R(6)$. In that case, he believes, we should attempt to destroy the aliens.*"

There is an alternative useful way to think about 22, by taking $\log_2$ of each bound and rearranging, we get that

$$\left(\frac{1}{2} + o(1)\right)\log_2 n \leq \min_{G=(V,E),\,|V|=n} r(G) \leq (2 + o(1))\log_2 n$$

The current "world record" (see [CZ15, Coh15]) for deterministic construction of families of graphs with small $r(G)$ achieves $r(G) \lesssim 2^{(\log\log|V|)^c}$, for some constant $c > 0$. Note that this is still considerably larger than $\mathrm{polylog}|V|$. In contrast, it is very easy for randomized constructions to satisfy $r(G) \leq 2\log_2 n$, as made precise by the folloing theorem.

References

[CZ15] E. Chattopadhyay and D. Zuckerman. Explicit two-source extractors and resilient functions. *Electronic Colloquium on Computational Complexity*, 2015.

[Coh15] G. Cohen. Two-source dispersers for polylogarithmic entropy and improved ramsey graphs. *Electronic Colloquium on Computational Complexity*, 2015.

[Spe94] J. Spencer. *Ten Lectures on the Probabilistic Method: Second Edition.* SIAM, 1994.

[9]By $a_k \lesssim b_k$ we mean that there exists a constant c such that $a_k \leq c\, b_k$.