

## Final Exam

### 2.003 Dynamics and Vibrations

This is a closed book quiz. Each student may bring three sides of an  $8\frac{1}{2}$  x 11 inch sheet of self-prepared notes.

#### Problem 1 (10pts)

A rigid, uniform cylinder of mass  $m$  and radius  $r$  is rolling down an incline with angle  $\theta$  as shown in figure 1. Determine the minimum value of the coefficient of static friction which will prevent slipping.

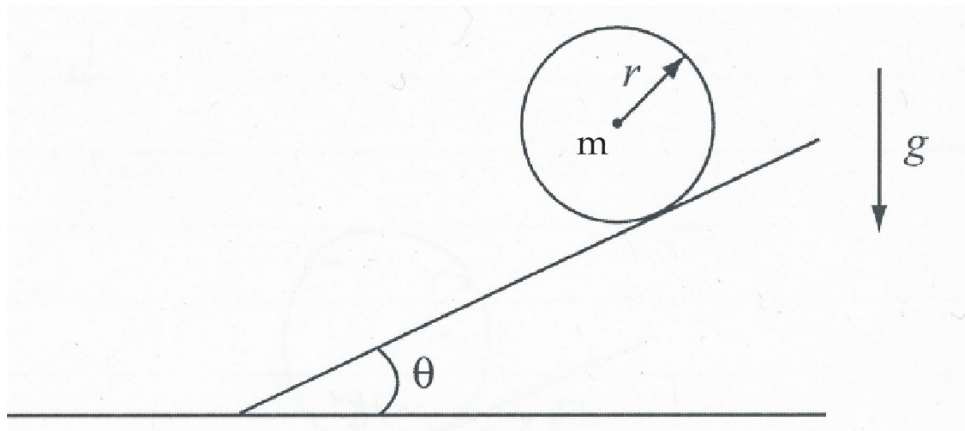


Figure 1

**Problem 2** (10 pts)

Figure 2 shows two pendulums of length  $l$  and masses  $m_1$  and  $m_2$  connected by a spring with spring constant  $k$ . The spring is unstretched when the two pendulums are hanging vertically at  $\theta_1 = \theta_2 = 0$ . Find the natural frequencies and the corresponding normal modes for small motions about equilibrium.

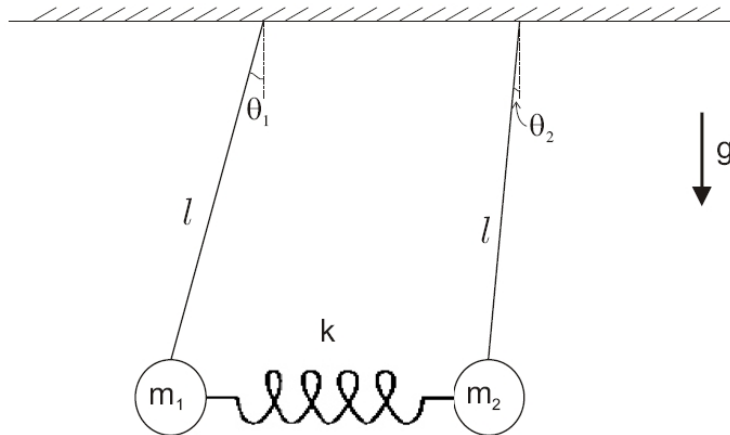


Figure 2

**Problem 3** (10 pts)

Two small disks of mass  $m$  are placed on a massless rod of length  $L$  as shown in figure 3. The rod is hung via a frictionless pivot on the ceiling. One small disk is affixed at the end of the rod. The second small disk can ride freely along the rod. The two disks are connected by a spring with spring constant  $k$ . Let  $d$  be the equilibrium separation between the two disks when rod is at rest hanging straight down.

Assume small motions around this equilibrium configuration and determine linearized equation(s) of motion.

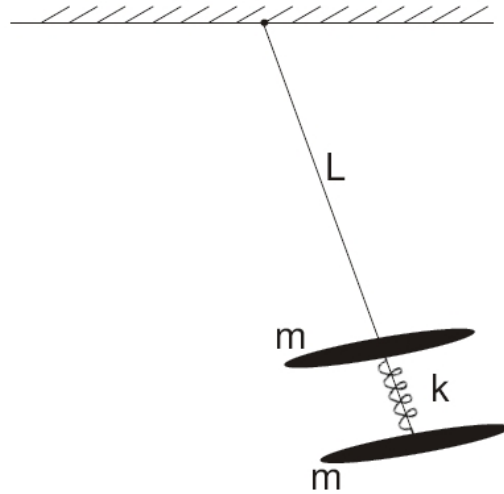


Figure 3

**Problem 4** (10 pts)

The uniform bar  $AB$  of mass  $M$  and length  $l$  has a small massless roller at each end and moves on a circular track as shown in figure 4. Using  $\theta$  as a generalized coordinate derive the equation of motion in the absence of friction. The length of  $OG$  is  $R$ . Using this equation of motion answer the following two questions concerning small motions about  $\theta = 0$ .

- Find the frequency of small oscillations about  $\theta = 0$ .
- Assume that this device is submerged in oil which exerts a significant drag force (parallel to  $AB$ ) given by  $-b\vec{v}_G$ , where  $b$  is a proportionality coefficient and  $\vec{v}_G$  is the velocity of point  $G$ . Find the values of  $b$  for which the bar will overshoot  $\theta = 0$  if released from rest from  $\theta \neq 0$ .

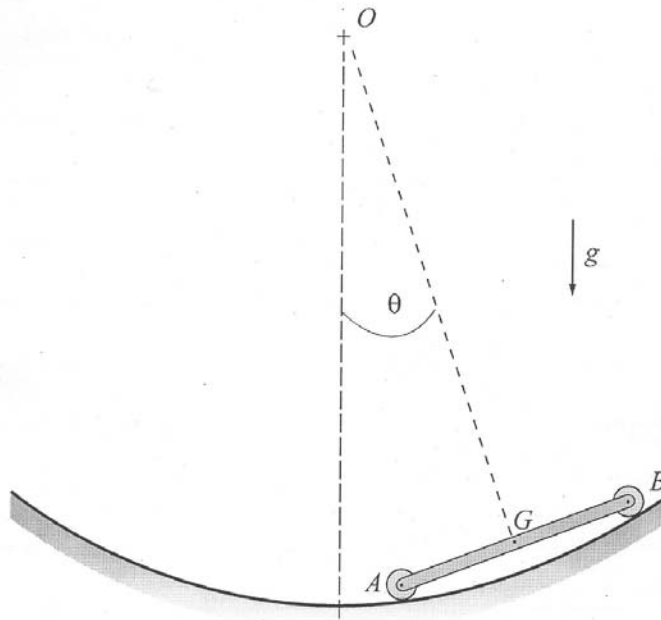


Figure 4

**Problem 5** (10pts)

Masses  $m$  and  $2m$  are attached by a spring with constant  $k$  as shown in figure 5. Sketch the frequency response diagrams for  $x_1$  and  $x_2$  when the mass  $2m$  is subject to an external force  $F_0 \cos(\omega t)$ .

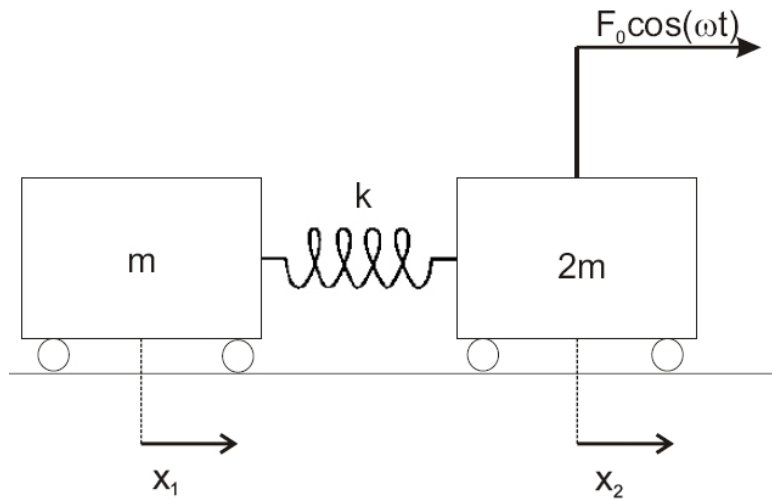


Figure 5