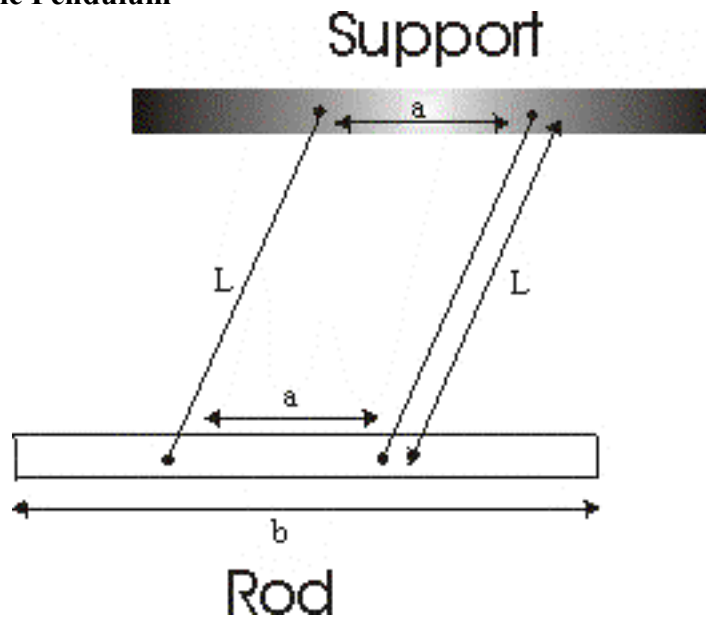# 2.004: MODELING, DYNAMICS, & CONTROL II
Spring Term 2003

PLEASE ALSO NOTE THAT ALL PRELAB EXERCISE ARE DUE AT THE START (WITHIN 10 MINUTE) OF THE LAB SESSION, NO LATE WORK IS ACCEPTED.

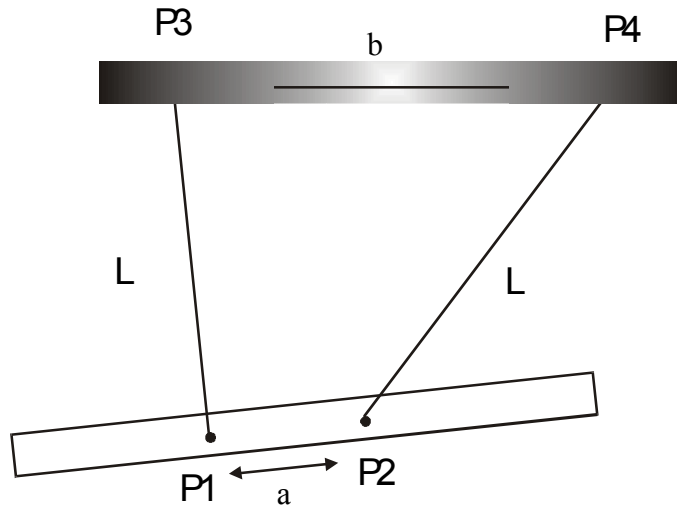**Pre-Lab Exercise for Experiment 3**

(1) Symmetric Pendulum

(a) Consider a rod pendulum with length b, mass m, suspended by two tethers with length L as shown above. The tethers are separated by distance, a, at the support surface and at the rod. Write down the equation of motion for the center of mass of the rod and deduce its natural oscillation frequency in the absence of loss mechanisms. You may neglect the mass of the tethers.

(b) Write an equation describing the trajectory of the center of mass as a function of time.

(c) From the solution in part (b), write an equation describing the trajectories of one end of the rod (please pick any one of the ends).

(2) Asymmetric Pendulum

Consider a rod pendulum with mass m, suspended by two tethers with length L as shown above. The tethers are separated by distance, b, at the support surface and separated by distance, a, at the rod. You may neglect the mass of the tethers.

(a) Explain why this system has only one degree of freedom?

(b) Consider the case where a=b and deduce the natural frequency of the pendulum.

(c) Now consider the more difficult case where a is differ from b. Since this system has only one degree of freedom, its kinematics can be described by one generalized coordinate. We can simplify the representation of the geometry of the system at a particular configuration as follows:

P3 b P4
$\theta 1$ $\theta 2$
L L
θ P2
P1 a
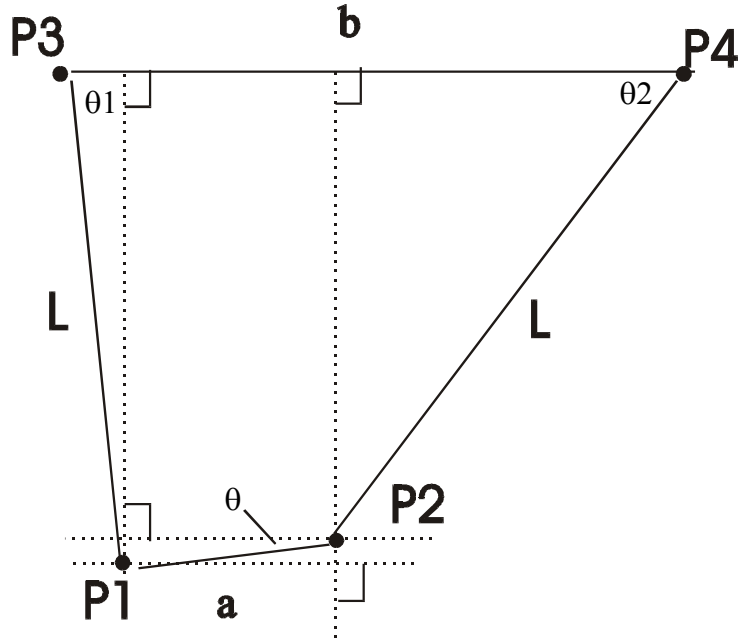
Choose θ_1 as the generalized coordinate, express the relationship between θ, θ_1, θ_2.

(d) At resting configuration, the values of θ_1 and θ_2 are equal (by symmetry) and we will designate the equilibrium values of the angles θ_1 and θ_2 as θ_0. Express θ_0 your result in terms of a, b, and L. Redraw the figure at equilibrium.

(e) Consider small perturbation of this pendulum from its equilibrium position such that:

$$\theta_1 = \theta_0 + \Delta\theta_1$$

$$\theta_2 = \theta_0 - \Delta\theta_2$$

where $\Delta\theta_1$, $\Delta\theta_2$ are assumed to be small. Further, you are given the Taylor expansions:

$$\sin(\theta_0 \pm \Delta\theta_i) = \sin\theta_0 \pm \Delta\theta_i \cos\theta_0$$
$$\cos(\theta_0 \pm \Delta\theta_i) = \cos\theta_0 \mp \Delta\theta_i \sin\theta_0$$

Verify that $\Delta\theta_1$ and $\Delta\theta_2$ are equal?

(f) In reference to the center of mass, force and moment balance equations in terms of $\Delta\theta_1$.

(g) Simply the equations of motion as a function of $\Delta\theta_1$. Express the natural frequency of this pendulum in terms of the gravitation constant g, geometric constants a, b, and L.