MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

2.004 Dynamics and Control II
Fall 2007

Problem Set #9

Posted: Friday, Nov. 16, '07 **Due: Wednesday, Nov. 28, '07**

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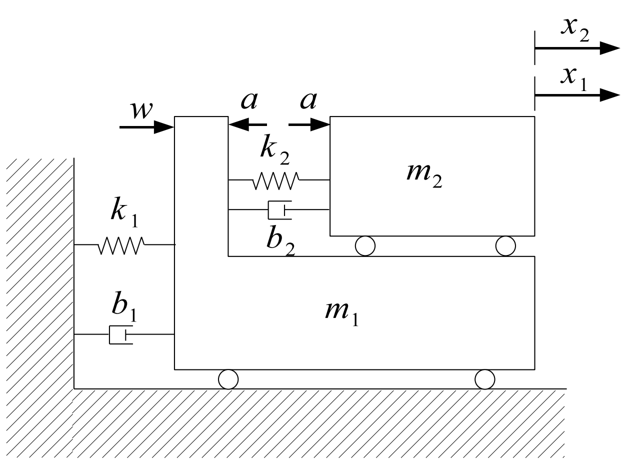
1. In this problem, we will analyze the 2.004 Tower system (including the compensating mass–damper–compliance slider components) using the eigenvalues/eigenvectors of its state matrix. The system parameters are $m_1 = 5.11\,\text{kg}$, $b_1 = 0.767\,\text{N}\cdot\text{sec/m}$, $k_1 = 2024$ N/m; $m_2 = 0.87$ kg, $b_2 = 8.9$ N $\cdot$ sec/m, $k_2 = 185$ N/m. The system model schematic is shown again below for your convenience.



a) Substitute these values into the system matrix **A** that you derived in Problem Set 8 (or use the solution to Problem Set 8 that has been posted online if you haven't kept a copy of your solution) and use MATLAB to compute the eigenvectors and eigenvalues as follows: `[va,da]=eigs(a)`. This will return two matrices `va` and `da`. The *columns* of matrix `va` are the eigenvectors; the *diagonal elements* of matrix `da` are the eigenvalues corresponding to the eigenvectors column–by–column.

b) Use the eigenvalues to justify the following statement: "The 2.004 Tower has two modes of oscillation, one slow and one fast." Compute the damped and natural frequencies of oscillation of the two modes.

c) Use the correspondence between eigenvectors and eigenvalues to justify the following statement: "In the slow mode the tower and slider mass oscillate in phase, while in the fast mode the tower and slider mass oscillate out of phase."

d) In MATLAB, define the matrices `b`, `c1` such that `b` is the actuation matrix with the wind force acting as the sole input to the system and `c1` is the observation matrix with the tower displacement as the system output. Also define a scalar `d=0`. Use the command `tower1=ss(a,b,c1,d)` to obtain the state–space representation of the system. Call the LTI Viewer, import `tower1` and select the impulse response. Which mode has been excited by the impulse input? Compare the damped frequency of oscillation of the mode that you think has been excited with the frequency of oscillation that you measure from the impulse response simulation.

e) Now define a new matrix `c2` such that the slider displacement is the system output, and a new state–space representation `tower2=ss(a,b,c2,d)`. Open a new LTI Viewer window (without closing the window that you generated in the previous question), import `tower2` and generate its impulse response. Is the phase relationship between the `tower1` and `tower2` responses consistent with the oscillation mode that the system is in?

2. Nise chapter 12, problem 4 (page 777).

3. Nise chapter 9, problem 1 (page 674).

4. Nise chapter 9, problem 2 (page 674). Use MATLAB to generate the plots.

5. Nise chapter 9, problem 4 (page 674).

6. For the following transfer function

$$G(s) = \frac{(s+10)}{(s+1)(s^2+40s+10^4)}$$

sketch the Bode asymptotic magnitude and phase plots, using appropriate corrections, and compare with the exact result using MATLAB.