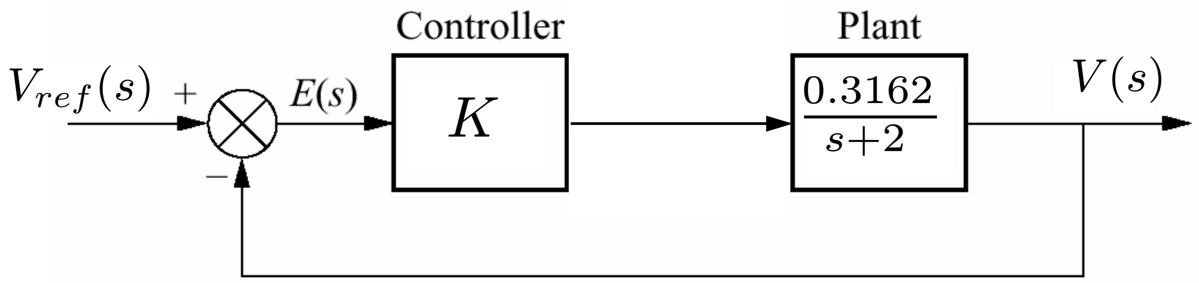
Cranking up the gain ☺

Type **0** system (no disturbance)

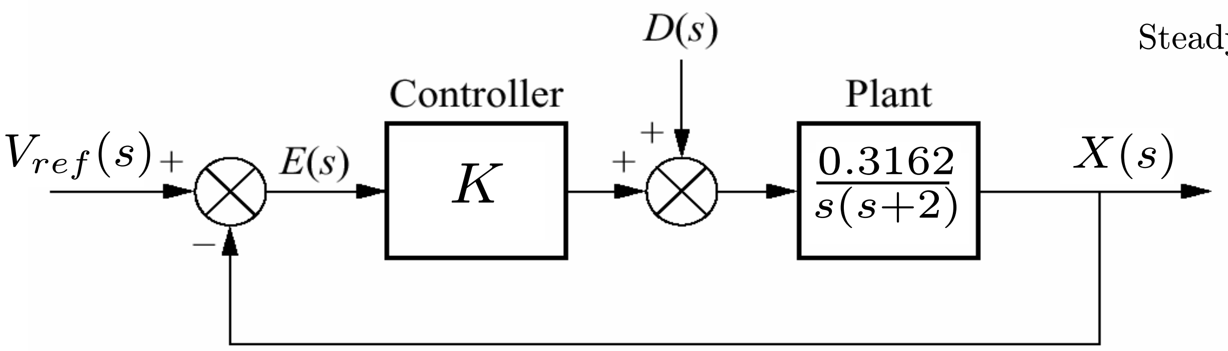


Steady–state error due to step input:

$$e_R(\infty) = \frac{2}{2 + 0.3162K}$$

$$e_R(\infty) \to 0 \quad \text{as} \quad K \to \infty$$

Type **1** system with disturbance



Steady–state error due to step input:

$$e_R(\infty) = 0$$

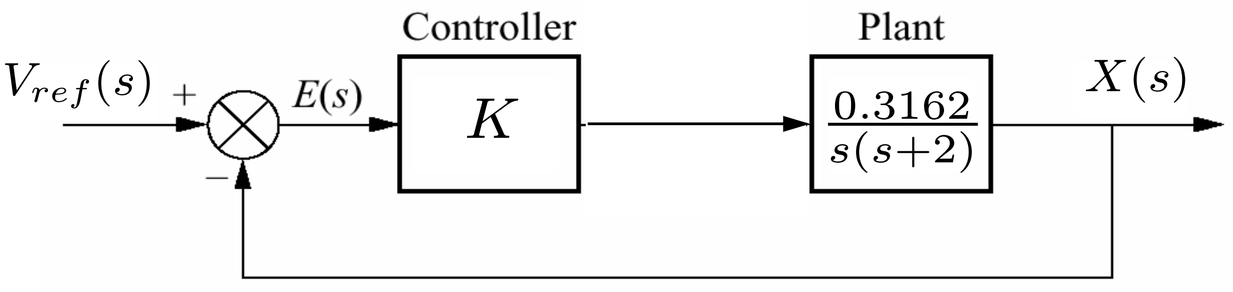
Steady–state error due to step disturbance:

$$e_D(\infty) = -\frac{1}{K}$$

$$e_D(\infty) \to 0 \quad \text{as} \quad K \to \infty$$

Cranking up the gain ☹

Type **1** system (no disturbance)



Closed–loop transfer function

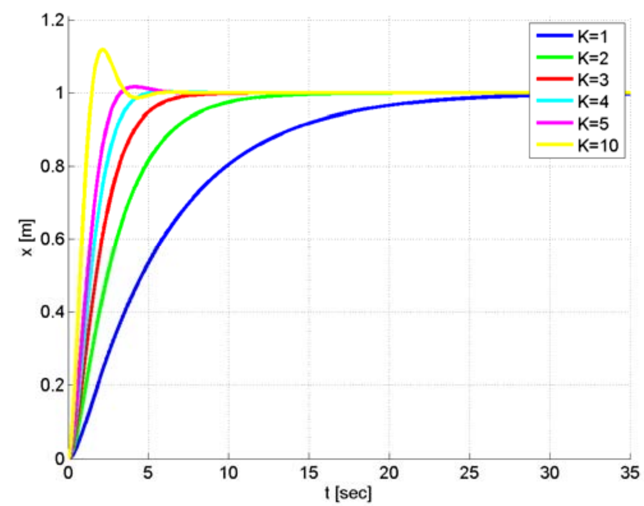
$$\frac{X(s)}{V_{ref}(s)} = \frac{0.3162K}{s^2 + 2s + 0.3162K}$$

Pole locations

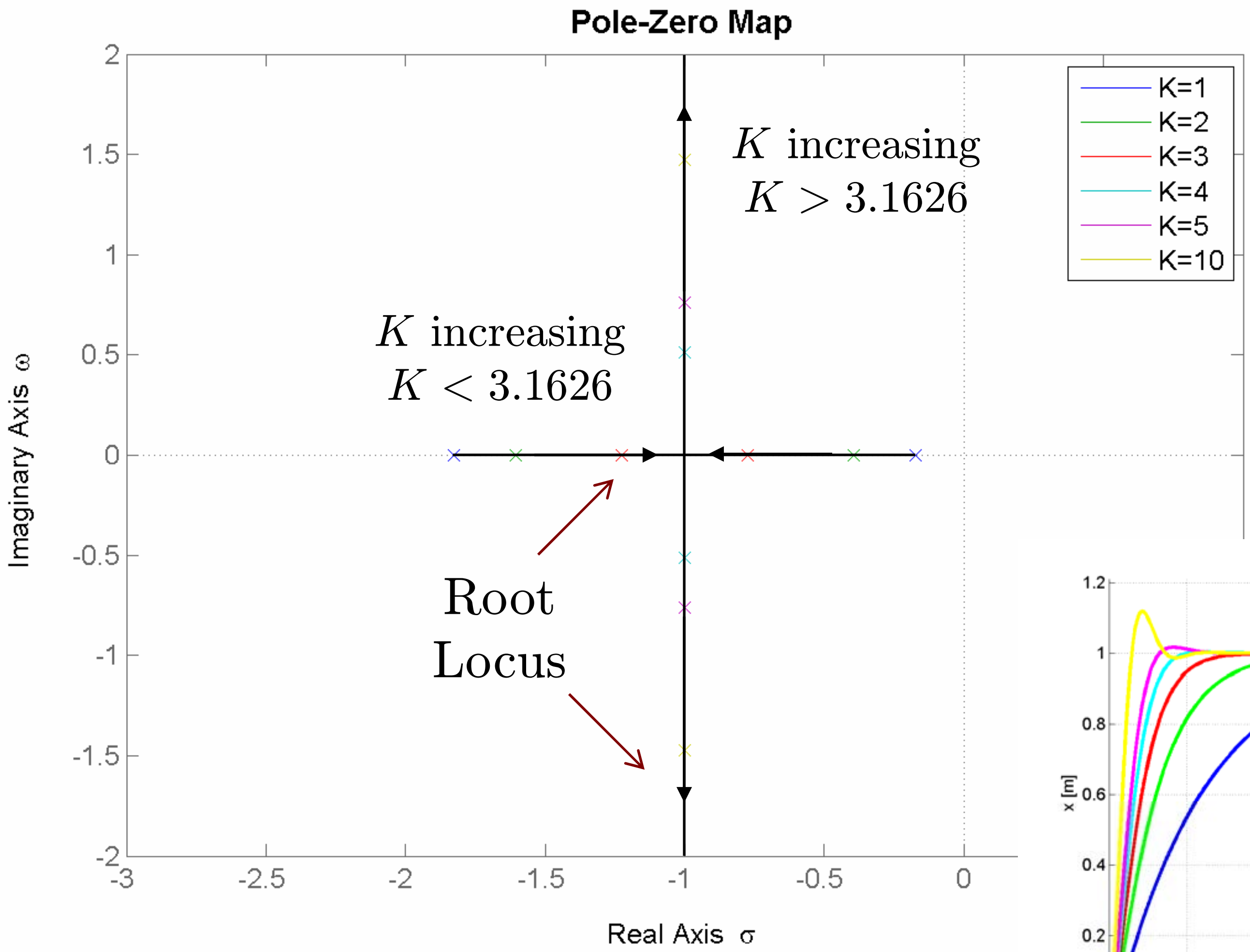
$$p_1 = -1 + \sqrt{1 - 0.3162K} \qquad p_2 = -1 - \sqrt{1 - 0.3162K}$$

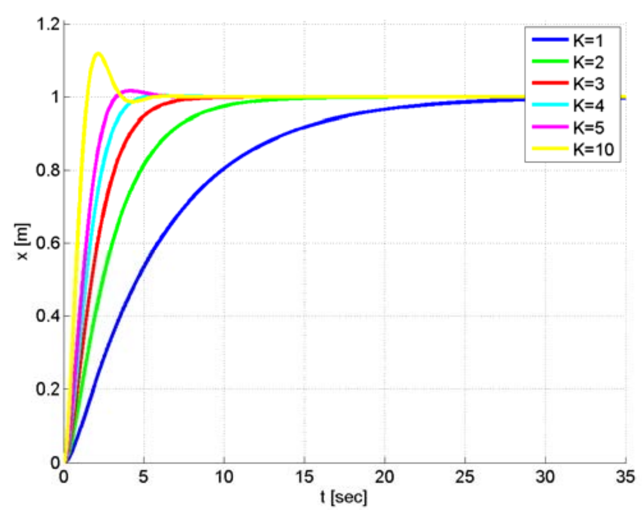
System becomes **underdamped**$\Rightarrow$
$\Rightarrow$ step response **overshoots** if

$$1 - 0.3162K < 0 \Leftrightarrow K > 3.1626$$

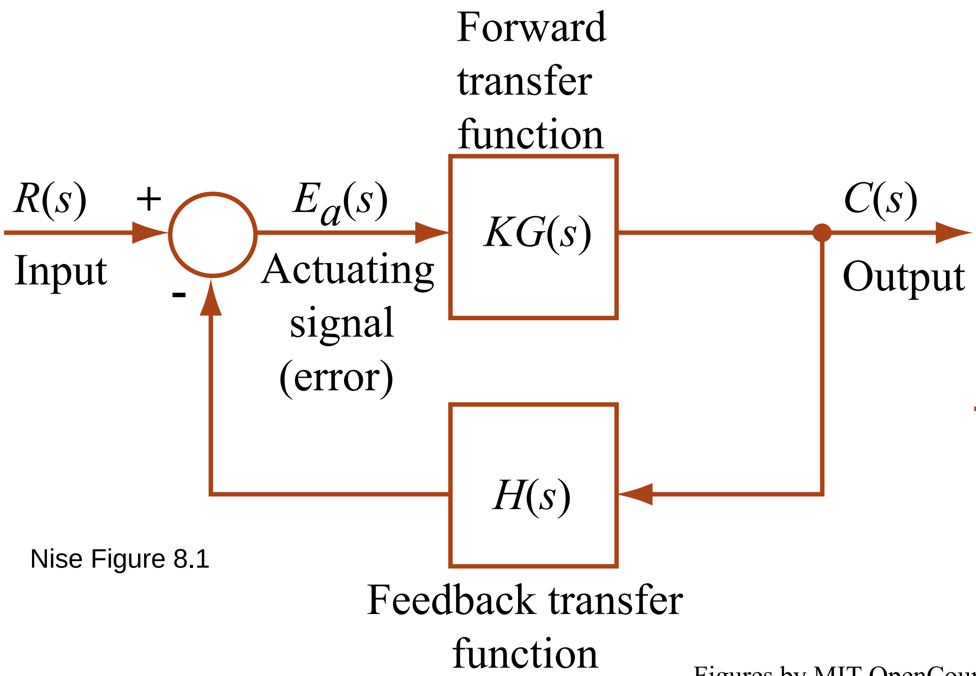


Cranking up the gain: poles and step response





Root locus for nonunity feedback systems

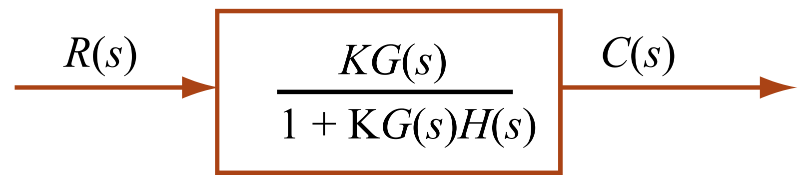


Nise Figure 8.1

Figures by MIT OpenCourseWare.

Caveat: $K>0$

Closed loop TF:



“Open loop” TF:
$G(s)H(s)$

Closed-loop pole locations

$$1+KG(s)H(s)=0 \Rightarrow \begin{cases} K=1/\left|G(s)H(s)\right|; \\ \angle KG(s)H(s)=(2n+1)180^\circ. \end{cases}$$

Root locus terminology

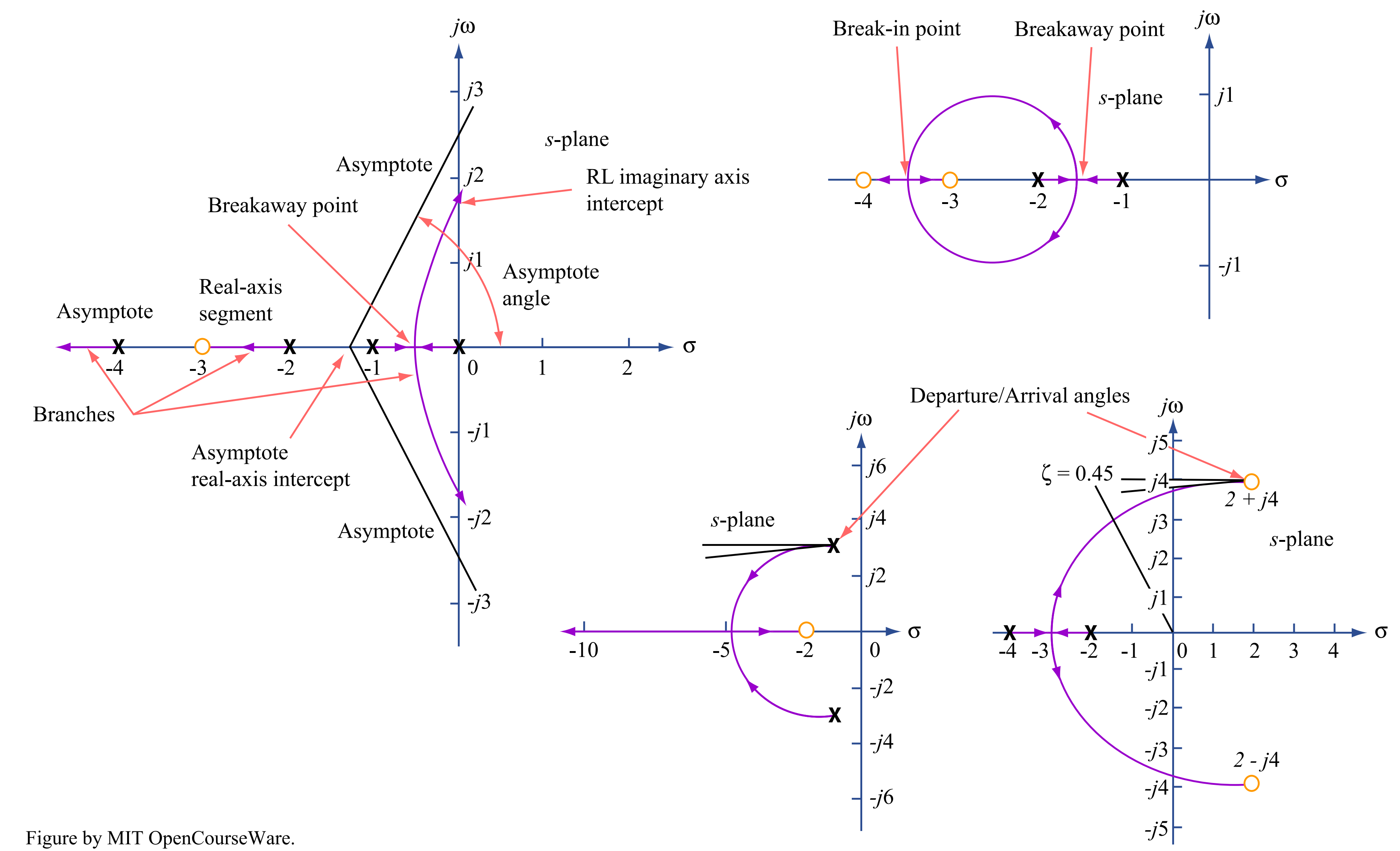


Figure by MIT OpenCourseWare.

Root-locus sketching rules

- **Rule 1:** # branches = # poles
- **Rule 2:** symmetrical about the real axis
- **Rule 3:** real-axis segments are to the left of an *odd* number of real-axis finite poles/zeros

$$\text{Let} \quad G(s) = \frac{N_G(s)}{D_G(s)}, \quad H(s) = \frac{N_H(s)}{D_H(s)}.$$

$$\Rightarrow \quad \angle G(s)H(s) = \sum \angle(\text{poles}) - \sum \angle(\text{zeros}).$$

Recall angle condition for closed–loop pole:

$$\angle KG(s)H(s) = (2n+1)180^\circ.$$

Complex–pole/zero contributions: **cancel** because of symmetry
Real–pole/zero contributions: each is 0° from the left, 180° from the right; total contributions from right must be odd number of 190°'s to satisfy angle condition.

Image removed due to copyright restrictions.

Please see: Fig. 8.8 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Root-locus sketching rules

- **Rule 4:** RL begins at poles, ends at zeros

$$\text{Let} \quad G(s) = \frac{N_G(s)}{D_G(s)}, \quad H(s) = \frac{N_H(s)}{D_H(s)}.$$

$$\Rightarrow \quad \text{Closed–loop TF}(s) = \frac{K N_G(s) D_H(s)}{D_G(s) D_H(s) + K N_G(s) N_H(s)}.$$

If $K \to 0^+$ (small gain limit)

$$\text{Closed–loop TF}(s) \approx \frac{K N_G(s) D_H(s)}{D_G(s) D_H(s) + \epsilon} \Rightarrow$$

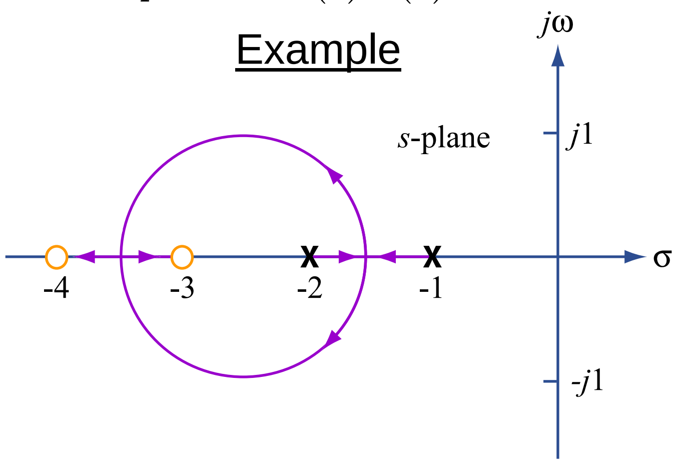
closed–loop denominator is *denominator* of $G(s)H(s)$
$\Rightarrow$closed–loop poles are the *poles* of $G(s)H(s)$.

If $K \to +\infty$ (large gain limit)

$$\text{Closed–loop TF}(s) \approx \frac{K N_G(s) D_H(s)}{\epsilon + K N_G(s) N_H(s)} \Rightarrow$$

closed–loop denominator is *numerator* of $G(s)H(s)$
$\Rightarrow$closed–loop poles are the *zeros* of $G(s)H(s)$.

<u>Example</u>



Nise Figure 8.10

Figure by MIT OpenCourseWare.

Please see the following selections from

Mathworks, Inc. "Control System Toolbox Getting Started Guide."

http://www.mathworks.com/access/helpdesk/help/pdf_doc/control/get_start.pdf

Ch. 4, pp. 3-18