2.016 Homework #3

Prof. A. Techet; Fall 2005
Issued: September 27, 2005
Due: October 4, 2005

Problem 1: Unsteady Potential Flow and Added Mass

a. Derive added mass around a sphere. Hint: if you print out the Added Mass Derivation handout, the math is all done for you, and you can hand that worksheet in. All you have to do is draw a picture on the page illustrating how to set up the integral (i.e. a picture of a sphere showing the coordinate system, $\vec{F}, p, d\vec{A}, R, r$ and θ, and another picture showing how to calculate $d\vec{A}_x$ from $d\vec{A}$.

b. In this derivation, are we integrating the static pressure or the dynamic pressure? What happens if we include the other pressure term as well?

c. The equation sheet says that the potential function for flow around a stationary sphere is $\phi = U\cos\theta\left(r + \frac{R^3}{2r^2}\right)$, but the potential function given in the derivation is $\phi = U\cos\theta\left(\frac{R^3}{2r^2}\right)$. What is the difference between these two potential functions? What type of flow does this correspond to? Why would we not include that in the derivation? (Hint: think about the answer to part d.)

d. True or false, $\phi = U\cos\theta\left(\frac{R^3}{2r^2}\right)$ corresponds to flow around a moving sphere in quiescent fluid?

e. Derive added mass around a cylinder. Just draw a picture on the page illustrating how to set up the integral (i.e. a picture of a sphere showing the coordinate system, $\vec{F}, p, d\vec{A}, R, r$ and θ, and another pic showing how to calculate $d\vec{A}_x$ from $d\vec{A}$.

f. In this derivation, are we integrating the static pressure or the dynamic pressure? What happens if we include the other pressure term as well?

g. In class, we learned that the potential function for flow around a stationary cylinder is $\phi = U\cos\theta\left(r + \frac{R^2}{r}\right)$, but the potential function given in the derivation is $\phi = U\cos\theta\left(\frac{R^2}{r}\right)$. What is the difference between these two potential functions? What type of flow does this correspond to? Why would we not include that in the derivation? (Hint: think about the answer to part d.)

h. True or false, $\phi = U\cos\theta\left(\frac{R^2}{r}\right)$ corresponds to flow around a moving cylinder in quiescent fluid?

Problem 2: Pressure Distribution Around a Stationary Cylinder

a. The potential function for flow around a stationary cylinder is $\phi = U\cos\theta\left(r + \frac{R^2}{r}\right)$, where U is the free-stream velocity far away from the cylinder. Find the velocity field $\vec{V}(r,\theta) = \frac{\partial\phi}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{e}_\theta$.

b. Find the pressure at the surface of the cylinder, $(r = R)$, using Bernoulli's equation, making use of the fact that the pressure at the stagnation points (where V=0) is the stagnation pressure, p_s.

c. Show that the coefficient of pressure, $c_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$, can be expressed in the form $c_p = 1 - \frac{V^2}{U^2}$.

d. Plot the coefficient of pressure using Matlab or Excel for $0 < \theta < 2\pi$.

Problem 3: Added Mass

Calculate the added mass coefficients m_{33} and m_{44} for a circular cylinder of radius $R = 1cm$ and length $L = 1m$ whose axis is along the 1 axis.

Problem 4: Added Mass

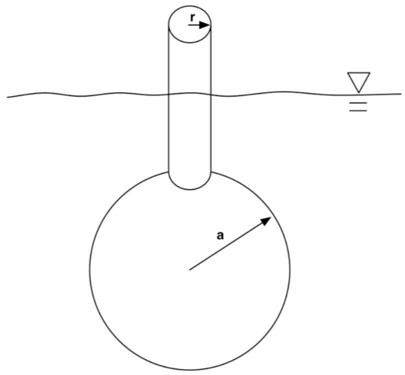
In lab, we discussed how added mass affects the natural frequency of a cylinder bobbing up and down under water.

a. Write the equation for the natural frequency for this underwater spring-mass system.

b. What do you expect to happen if the cylinder has a square cross section?

Problem 5: Buoyancy and Added Mass

A buoy consists of a large sphere of radius, a, under a circular cylinder of radius, r:



The added mass of the cylinder is negligible compared to that of the sphere:

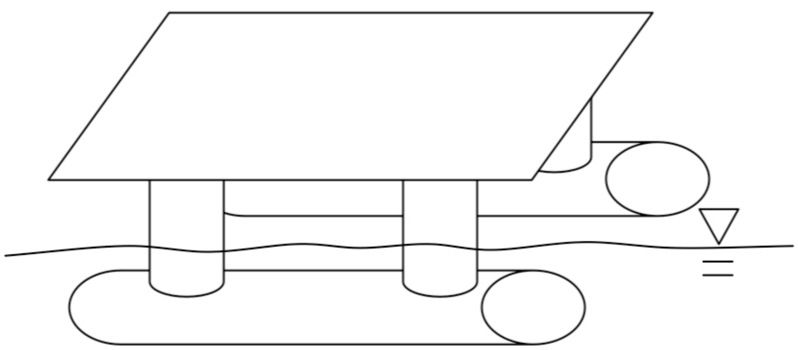
a. Write the equation of motion for heave. (Note: there is a force that increases linearly with depth.)

b. What is the buoy's natural frequency in heave.

Problem 6: Buoyancy and Added Mass

An offshore platform has the configuration shown:

(both pontoons underwater)



The diameter of the uprights is 10 m, and that of the pontoons is 10 m. The length of the pontoons is 100 m. The added mass of the uprights is negligible compared to that of the pontoons.

a. Write the equation of motion for heave.

b. What is the platform's natural frequency in heave.