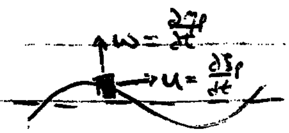
# 2.016 HW #6 Soln

1\. a) <u>Wind</u> is the driving force for most ocean waves.

b) 

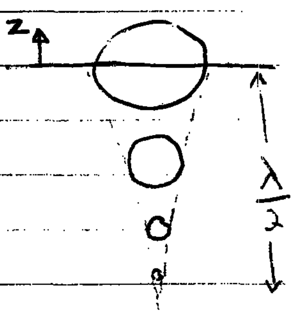

The fluid particle follows the local velocity at each point in space.

<u>Lagrange says:</u> $\frac{\partial \xi_p}{\partial t}$ is the x velocity of a fluid particle

<u>Euler says:</u> $u$ is the local x velocity at each point in space.

When we talk about "motion of the free surface" we think about the velocity of fluid elements on the free surface. When we talk about the potential function and the velocity of a wave ($u = \frac{\partial \phi}{\partial x}$) we are describing the velocity of the entire <u>flow field</u>. At the surface, the motion of the fluid element $\frac{\partial \xi_p}{\partial t}$ <u>must be the same</u> as what we prescribe for $u$.

↳ By understanding the Lagrangian and Eulerian points of view, we can know when to use one or the other or <u>set them equal</u>.

c) 

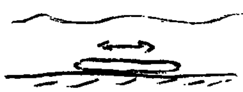

$$u = a\omega \frac{\cosh(k(z+H))}{\sinh(kH)} \cos(kx-\omega t)$$

$$= \frac{\cosh(kz)\cosh(kH)}{\underbrace{\sinh(kH)}_{=1 \text{ for } kH\to\infty}} + \frac{\sinh(kz)\cancel{\sinh(kH)}}{\sinh(kH)}$$

$$= \cosh(kz) + \sinh(kz) \quad \left( = \frac{e^x+e^{-x}}{2} + \frac{e^x-e^{-x}}{2} = e^x \right)$$

$$= e^{kz}$$

$$e^{kz} \approx 0 \text{ when } -z > \frac{\lambda}{2} = \frac{\pi}{\omega} \qquad e^{-\pi} = 0.04$$

d) 

The scuba diver is pushed back & forth as the waves pass.

2. <u>linear</u> if $\boxed{\frac{2a}{\lambda} < \frac{1}{7}}$ the wave height is less than one seventh of the wavelength

<u>shallow</u> if $\boxed{\frac{H}{\lambda} < \frac{1}{20}}$ $\Rightarrow$ $\omega = \sqrt{gH}\, k$ $\rightarrow$ $k = \frac{\omega}{\sqrt{gH}}$

<u>intermediate</u> if $\boxed{\frac{1}{20} < \frac{H}{\lambda} < \frac{1}{2}}$ $\Rightarrow$ $\omega^2 = gk \tanh(kH)$ $\rightarrow$ $k =$ solve iteratively

deep if $\boxed{\frac{1}{2} < \frac{H}{\lambda}}$ $\Rightarrow$ $\omega^2 = kg$ $\rightarrow$ $k = \frac{\omega^2}{g}$

| $\omega$ (rad/s) | $H$ (m) | $\Rightarrow$ solve dispersion relation for $k$ ($\frac{1}{m}$) | $\lambda = \frac{2\pi}{k}$ | $a$ (m) | $\frac{2a}{\lambda}$ | $1/7$ | Linear? |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 10.2 | 0.616 | 0.02 | 0.06 | 0.14 | yes |
| 10 | 0.1 | 12.2 | 0.517 | 0.06 | 0.23 | 0.14 | no |
| 2 | 100 | 0.41 | 15.4 | 5 | 0.65 | 0.14 | no |
| 1 | 100 | 0.10 | 61.6 | 2 | 0.06 | 0.14 | yes |
| 5 | 0.5 | 2.9 | 2.2 | 0.1 | 0.09 | 0.14 | yes |
| 4 | 0.5 | 2.1 | 3.0 | 0.05 | 0.03 | 0.14 | yes |

Using Matlab:

```
k = fzero( @(k) (9.81)*k*tanh(k*H) - w^2, 1)
```

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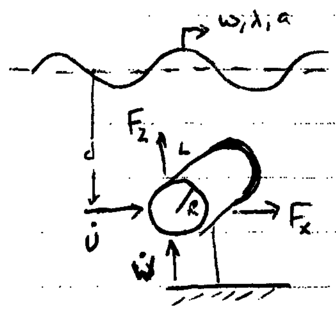
3.




For a linear free-surface wave:

$$\eta = a\cos(kx-\omega t)$$

$$u = a\omega f(z)\cos(kx-\omega t) \quad , \quad f(z) = \frac{\cosh(k(z+H))}{\sinh(kH)}$$

$$w = a\omega f_1(z)\sin(kx-\omega t) \quad , \quad f_1(z) = \frac{\sinh(k(z+H))}{\sinh(kH)}$$

$$\underline{F} = (m_a + \rho\forall)\,\underline{a}$$

$m_a = \rho\pi R^2 L$ for cylinder

$$\dot{u} = \frac{\partial u}{\partial t} = a\omega^2 f(z)\sin(kx-\omega t)$$

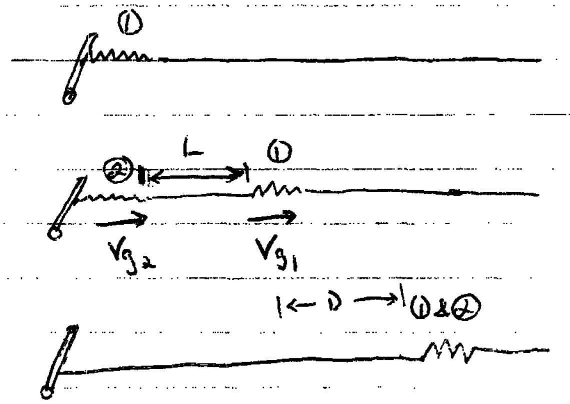
$$\dot{w} = \frac{\partial w}{\partial t} = -a\omega^2 f_1(z)\cos(kx-\omega t)$$

$$\boxed{\begin{aligned} F_x &= 2\rho\pi R^2 L \cdot a\omega^2 f(-d)\,\sin(kx-\omega t) \\ F_z &= -2\rho\pi R^2 L a\omega^2 f_1(-d)\cos(kx-\omega t) \end{aligned}}$$

4. time $t=0$

$t = \tau$

$t = t^*$




- The wave packets move at group speed, $V_g$

$$V_g = \frac{1}{2}\left(\frac{\omega}{k}\right)\left(1 + \frac{kH}{\sinh kH\cosh kH}\right)$$

$V_p$

$$V_{g_1} = \frac{L+D}{t^*} \rightarrow t^* = \frac{L+D}{V_{g_1}}$$

$$V_{g_2} = \frac{L+D}{t^*-\tau} \rightarrow L+D = V_{g_2}(t^*-\tau)$$

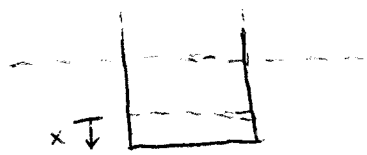
$$t^* = \frac{V_{g_2}}{V_{g_1}}(t^*-\tau)$$

$$\boxed{t^* = \left(\frac{1}{1-\frac{V_{g_1}}{V_{g_2}}}\right)\tau}$$

check: if $V_{g_1} = \frac{1}{2}V_{g_2}$, then $t^* = 2t$. ✓

5.

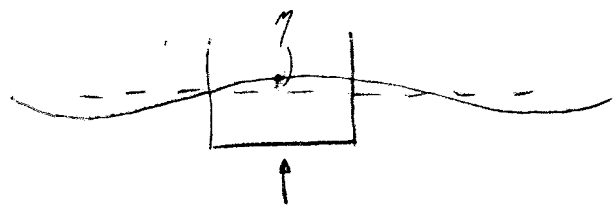
**Buoyancy**




$F_B = A_{wp} \cdot x$

- Buoyancy is a restoring force that opposes the ship's motion away from its equilibrium position. As the ship goes down, buoyancy pushes up.

**Froude-Krylov**




$F_{FK} = (\text{constants}) \cdot \eta$

- Froude-Krylov is a driving force that tries to pull the ship from its equilibrium position. As a wave passes, if the boat were fixed at its equilibrium position and a crest passes, it is like the boat is in deeper water, so a force pushes it up towards the surface, similar (hueristically) to buoyancy.

The equation of motion for the boat is:

$$A\ddot{x} + B\dot{x} + \underbrace{Cx}_{\text{Buoyancy is a restoring force.}} = \underbrace{F(t)}_{\text{Froude-Krylov is a driving force}}$$