40 Metacentric Height of a Catamaran

Consider the roll stability of a catamaran described as follows: total beam is $6b$, where b is the beam of each hull, the draft is T, and each hull is rectangular in cross-section.

1. What is the distance KM for this section, at small roll angles? You may approximate the area of the "buoyancy wedges" as the mean height times the width.

   The differential moment is as follows:

   $$dM = 2 \times \frac{5}{2}\, b^2\theta \times \frac{5}{2}\, b \times \rho g\ dx,$$

   where the terms separated by the $\times$ symbol are for: two wedges, the area enclosed in each wedge (approximation), the moment arm of each wedge, density and gravity and the differential length. We know that $\rho g d\nabla(KM)\theta = dM$, which leads directly to

   $$KM = \frac{25}{4}\frac{b^2}{T}.$$

   There is an additional term $T/2$ (as in the case of monohulls) but this is very small by comparison for a catamaran.

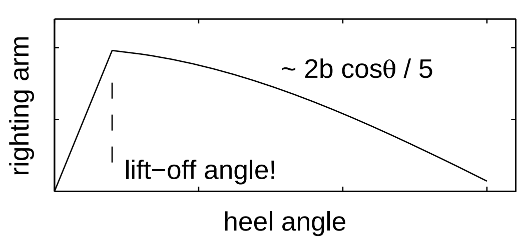
2. What is KM for a single block of draft T and beam $2b$?

   We find for the block $KM = b^2/3T + T/2$. The "wedge" term ($b^2/3T$) is about one-twentieth of the value from part a).

3. Make a sketch of the righting moment as the angle increases from zero to the point where one hull lifts out of the water, and then beyond. Describe what has happened in this situation and hence one of the inherent problems with catamarans.

   See the figure: The righting moment essentially goes flat once a hull is out of the water, because the there is no more loss of buoyancy on one side and gain of buoyancy on the other, as roll increases. You have left only the center of mass against buoyancy at the submerged hull. As the angle increases further, the righting moment arm decreases with the cosine of the roll angle.

   A proa is an unusual sailing boat with one large hull, and a very light pontoon on a long arm. Proas are double-ended - you keep the pontoon on the leeward side!



2.017J Design of Electromechanical Robotic Systems
Fall 2009