42   Submerged Body in Waves

A fixed, submerged body has a circular cross-section of constant diameter one meter, with length twenty meters. For the calculations below, use the Morison formulation for wave-induced force, with $C_d = 1.2$. Fix the wave amplitude at one meter, and consider the frequency range of 0.1 to two radians per second.

For each of the first four items below, in addition to making a plot please answer: What is the maximum magnitude and at what frequency does it occur? What are the physical mechanisms that create the curve you see?

1. Consider the body oriented horizontally and pointing into waves, at a fixed depth of four meters. Calculate and show in a graph the total heave force magnitude as a function of wave frequency ω.

   See Figure 1. The maximum magnitude of vertical force is 18.1kN, at about 1.1 rad/s. Since the wave amplitude is taken as constant at all frequencies, the Morison forces are increasing rapidly as the frequency goes up. At about 1.75 rad/s, however, there is a zero net force because the wavelength is the same as the body length.

   The fact that this plot is made for a unity wave amplitude means that the square of this function could be multiplied by the wave spectrum, to get the spectrum of the force. Beware of the deep-water assumption however, as indicated below.

2. Under the same conditions, calculate and show in a graph the total pitch moment magnitude as a function of wave frequency. Take the moment around the center of the body ($L/2$).

   The maximum pitch moment magnitude is 122 kNm at 1.45 rad/s. Explanation is similar to the case of vertical force, except that the zero frequency is now quite high; it is where the wavelength is equal to about sixty percent of the body length (not half!).

3. Now let the body be vertically oriented - e.g., it is a fixed piling. Using the **deep-water** wave formulas, calculate and show what is the magnitude of the total surge force on the piling, as a function of wave frequency.

   The maximum surge force magnitude seen is 16.7kN, at the maximum considered frequency of two radians per second. The forces get larger and larger because the velocities and accelerations at all depths both grow with frequency. This effect is stronger than the effect of the depth envelope getting narrower, over most of the frequency range. As above, also note that the wave amplitude is taken to be one meter for all frequencies.

4. Under the same conditions, calculate and show in a graph the pitch moment magnitude as a function of wave frequency. Take the moment around the lower end of the body.

   The maximum pitch moment magnitude seen is 292kNm, at the maximum considered frequency of two radians per second.

5. If the water depth is just twenty meters, for what range of frequencies is the analysis of the vertical body loading reasonable? In deep water, we approximate the wave velocity as $\sqrt{g/k}$, whereas in the general case it is $\sqrt{g\tanh(kh)/k}$, where h is the water depth.

Figure 2 shows that $\sqrt{\tanh(kh)}$ is close to one for all frequencies above one radian per second. Hence all of the maxima reported above are valid because they occur above this value, typical to SS 3. In practice, we have to design for higher sea states, which have lower frequency content as well as larger amplitudes; in these cases, we have to take into account the shallow-water formulae.
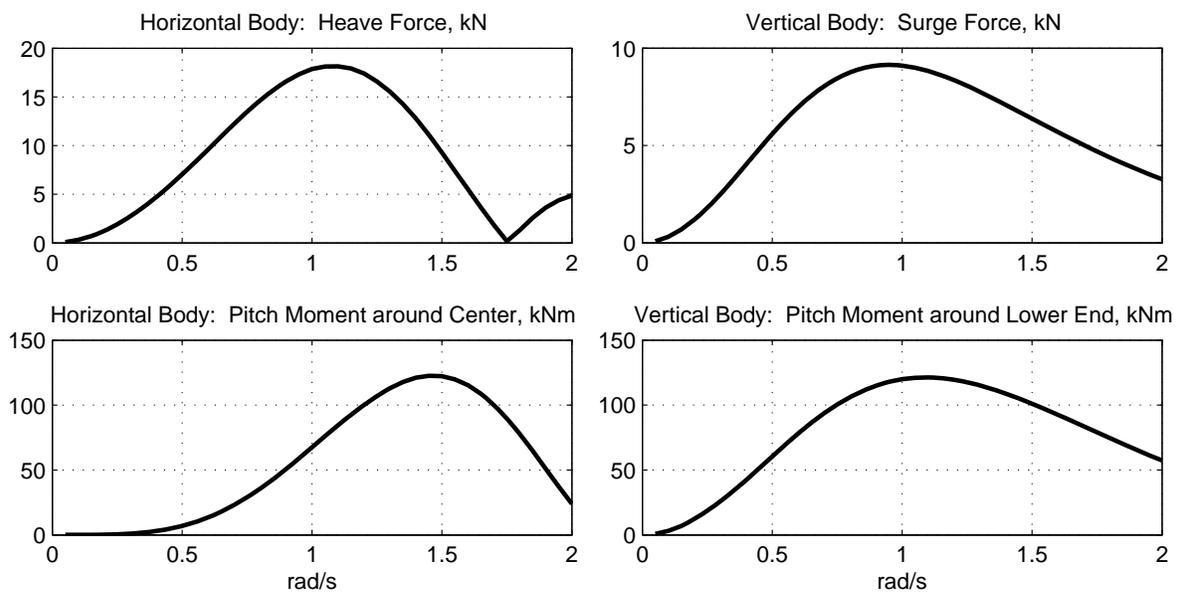

Figure 9: Force and moment on the horizontally-oriented body (left), and the vertically-oriented body (right).
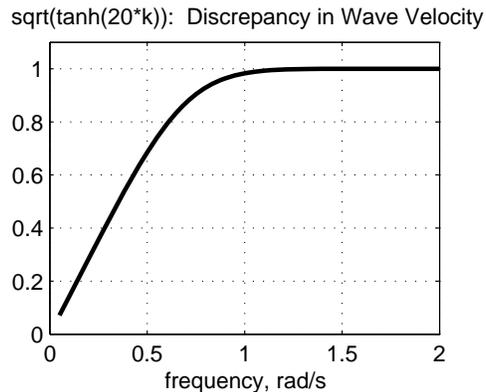
sqrt(tanh(20*k)): Discrepancy in Wave Velocity

Figure 10: The deep-water approximation is reasonable when this function is near one.

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%-----------------------------------------------------------
% Forces on a submerged body in waves

clear all;


d = 1 ; % cylinder diameter, m
L = 20.0 ; % cylinder length, m
Cd = 1.2 ; % Morison drag coefficient
T = 4 ; % depth of centerline, when the body is horizontal, m
dx = L/50 ; % differential length for strip theory calculations
g = 9.81 ; % gravity, m/s^2
rho = 1000 ; % water density, kg/m^3
A = 1 ; % wave amplitude, m

ntimes = 41 ; % number of time steps per cycle to study
wvec = .05:.05:2 ; % frequency vector, rad/s
xvec = dx/2:dx:L-dx/2 ; % body longitudinal location vector; centers

for i = 1:length(wvec) ; % loop through the frequencies of interest
    w = wvec(i) ;
    k = w^2/g ; % wave number
    tvec = 0:2*pi/w/(ntimes-1):2*pi/w ; % make up a one-cycle time
                                        % vector for this freq.

    for jj = 1:length(tvec), % loop through time
        t = tvec(jj) ;

        for j = 1:length(xvec) ; % loop through longitudinal locations
            x = dx*j ;
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            % first get the vertical velocity, acceleration, and
            % Morison force
            % on the horizontal body; note the Archimedes force!
            e1 = exp(-k*T);
            v1 = A*e1*(-w)*sin(w*t-k*x) ; % local vertical velocity
            vv1 = A*e1*(-w^2)*cos(w*t-k*x) ; % local vertical acc.
            dFz(j) = 1/2*rho*Cd*d*dx*v1*abs(v1) + ...
                rho*(pi/4*d^2)*2*dx*vv1 ; % differential force

            % now get the horizontal velocity, acceleration, and
            % Morison force on the vertical body; note the
            % Archimedes force!
            e2 = exp(-k*(L-x+4)) ;
                % (move from the seafloor to the surface)
            v2 = A*e2*w*cos(w*t) ;
                % local horiz. veolcity (horiz. position is zero,
                % so kx=0)
            vv2 = A*e2*(-w^2)*sin(w*t) ; % local horiz. acceleration
            dFx(j) = 1/2*rho*Cd*d*dx*v2*abs(v2) + ...
                rho*(pi/4*d^2)*2*dx*vv2 ;

        end;
        Fz(jj) = sum(dFz); % net vertical force on horiz. body
        Momz(jj) = sum(dFz.*(xvec-L/2));
            % pitch moment around horiz. body mid-point
        Fx(jj) = sum(dFx) ; % net horizontal force on vertical body
        Momx(jj) = sum(dFx.*xvec);
            % pitch moment around lower end of vert. body
    end;

    Fzf(i) = max(abs(Fz)) ;
        % get the force and moment magnitudes over the whole cycle
    Momzf(i) = max(abs(Momz));
    Fxf(i) = max(abs(Fx)) ;
    Momxf(i) = max(abs(Momx));

end;

figure(1);clf;hold off;
subplot('Position',[.2 .5 .4 .2]);
plot(wvec,Fzf/1000,'LineWidth',2);
title('Horizontal Body:  Heave Force, kN');
grid;
```

```
subplot('Position',[.2 .2 .4 .2]);
plot(wvec,Momzf/1000,'LineWidth',2);
xlabel('rad/s');
title('Horizontal Body:  Pitch Moment around Center, kNm');
grid;

figure(2);clf;hold off;
subplot('Position',[.2 .5 .4 .2]);
plot(wvec,Fxf/1000,'LineWidth',2);
title('Vertical Body:  Surge Force, kN');
grid;
subplot('Position',[.2 .2 .4 .2]);
plot(wvec,Momxf/1000,'LineWidth',2);
xlabel('rad/s');
title('Vertical Body:  Pitch Moment around Lower End, kNm');
grid;

% display the specific maxima
disp('----------------------------------------------------------------');
disp('Horizontal body:');
[junk,ind] = sort(Fzf) ;
disp(sprintf('Maximum heave force magnitude:  %g N at %g rad/s.',...
    Fzf(ind(end)),wvec(ind(end))));
[junk,ind] = sort(Momzf) ;
disp(sprintf('Maximum pitch moment magnitude:  %g Nm at %g rad/s.',...
    Momzf(ind(end)),wvec(ind(end))));
disp('Vertical body:');
[junk,ind] = sort(Fxf) ;
disp(sprintf('Maximum surge force magnitude:  %g N at %g rad/s.',...
    Fxf(ind(end)),wvec(ind(end))));
[junk,ind] = sort(Momxf) ;
disp(sprintf('Maximum pitch moment magnitude:  %g Nm at %g rad/s.',...
    Momxf(ind(end)),wvec(ind(end))));

% compute and plot the test function for deep-water wave approximation
kvec = wvec.^2/g ;
figure(3);clf;hold off;
subplot('Position',[.2 .2 .3 .3]);
plot(wvec,sqrt(tanh(kvec*L)),'LineWidth',2);
xlabel('frequency, rad/s');
title('sqrt(tanh(20*k)):  Discrepancy in Wave Velocity');
axis([0 2 0 1.1]);
grid;
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%------------------------------------------------------------------
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