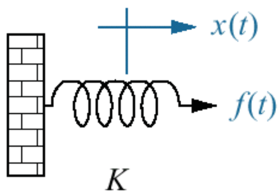
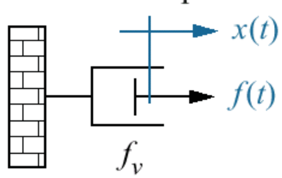
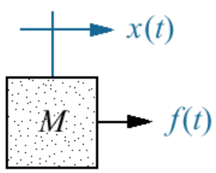
Goals for today

- Review of translational dynamical variables: position, velocity
- Review of rotational dynamical variables: angle, angular velocity
- Electrical dynamical variables: charge, current, voltage
- Basic electrical components
  - Resistance
  - Capacitance
  - Inductance
- DC Motor: an *electro-mechanical* element
  - basic physics & modeling
  - equation of motion
  - transfer function
- Experiment: step and ramp response of the flywheel driven by the DC motor open loop (no feedback)

Impedances: translational mechanical

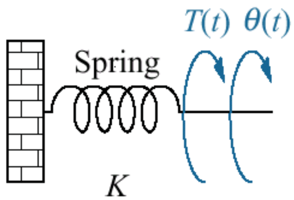
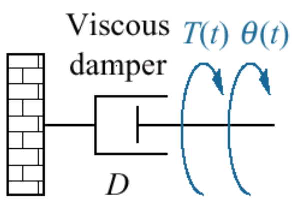
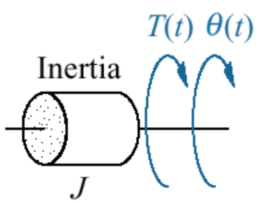
Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

(In the notes, we sometimes use b or B instead of f_v.)

Nise Table 2.4

Impedances: rotational mechanical

Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)\,d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

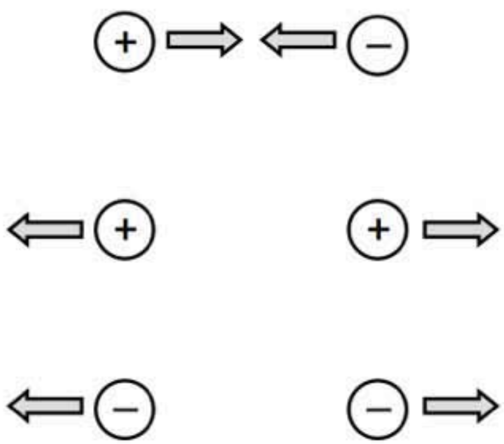
Note: The following set of symbols and units is used throughout this book: $T(t)$ = N-m (newton-meters), $\theta(t)$ = rad (radians), $\omega(t)$ = rad/s (radians/second), K = N-m/rad (newton-meters/radian), D = N-m-s/rad (newton-meters-seconds/radian), J = kg-m^2 (kilogram-meters^2 = newton-meters-seconds^2/radian).

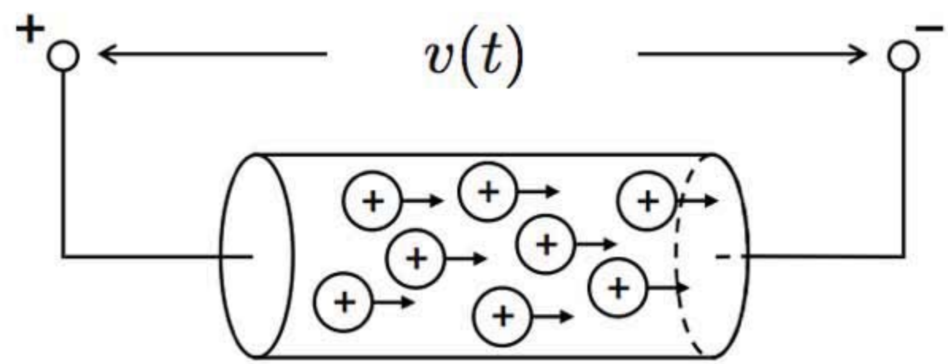
(In the notes, we sometimes use b or B instead of D.)

Nise Table 2.5

Electrical dynamical variables: charge, current, voltage

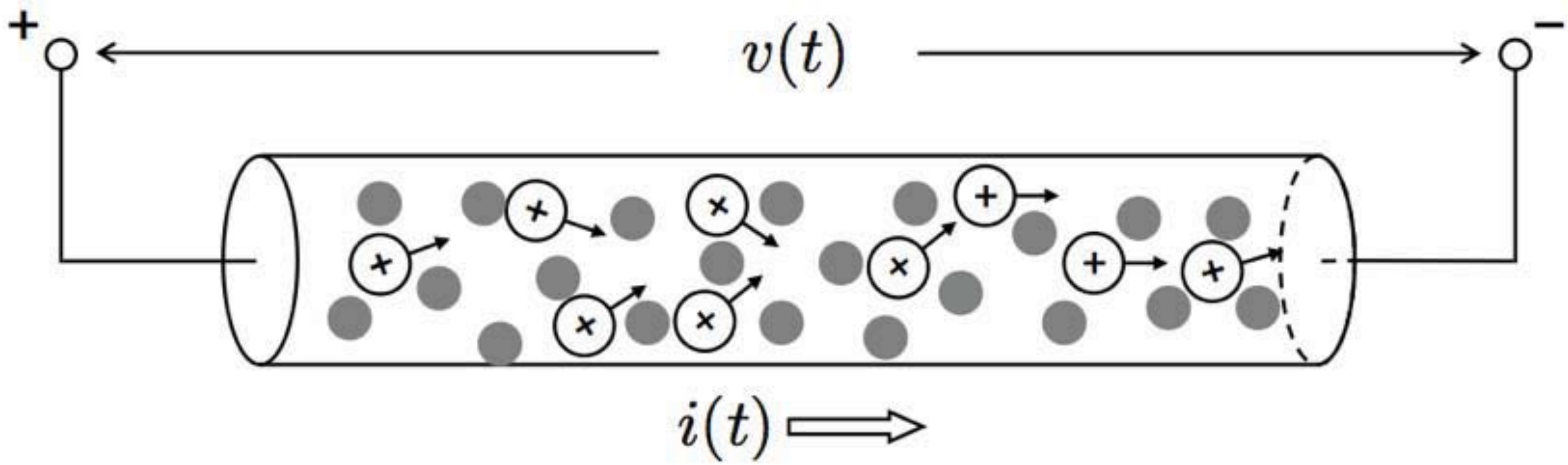
charge q	Coulomb [Cb]
charge flow $\equiv$ current $i(t)$	Ampére [A]=[Cb]/[sec]
voltage (*aka* potential) $v(t)$	Volt [V]





$$i(t) := \frac{\mathrm{d}q(t)}{\mathrm{d}t}$$

Electrical resistance

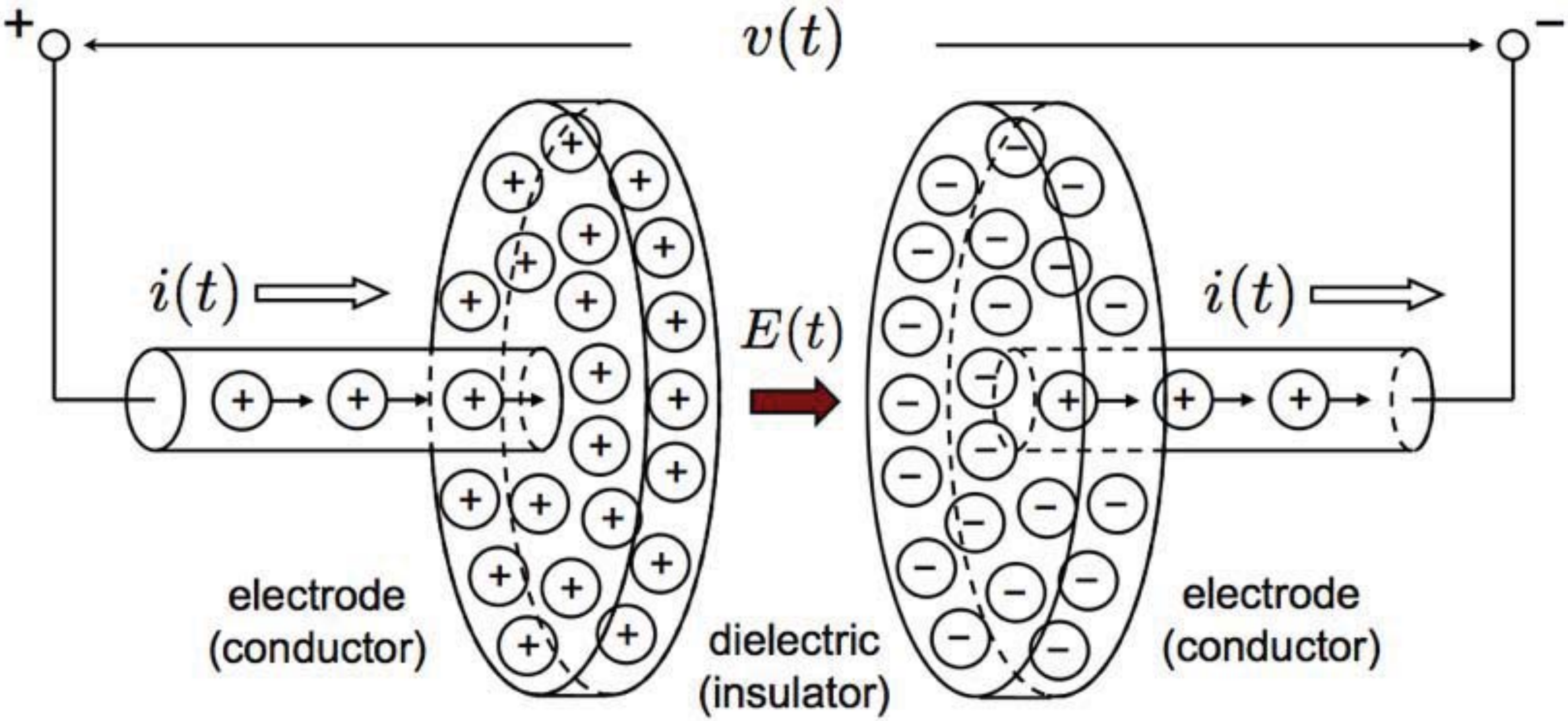


- Collisions between the mobile charges and the material fabric (ions, generally disordered) lead to <u>energy dissipation</u> (loss). As result, energy must be expended to generate current along the resistor; i.e., the current flow requires application of potential across the resistor

$$v(t) = Ri(t) \Rightarrow V(s) = RI(s) \Rightarrow \frac{V(s)}{I(s)} = R \equiv Z_R$$

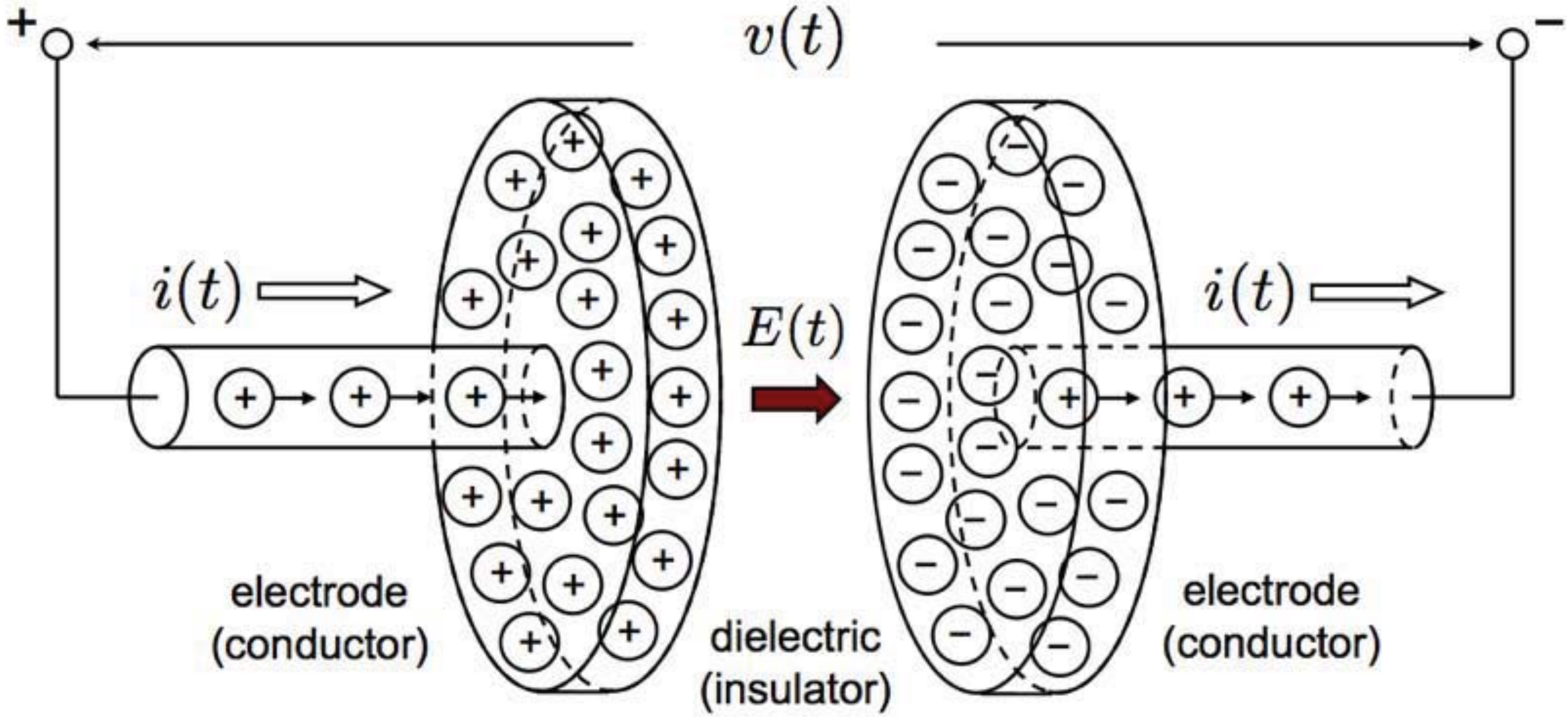
- The quantity Z_R=R is called the <u>resistance</u> (unit: Ohms, or Ω)
- The quantity G_R=1/R is called the <u>conductance</u> (unit: Mhos or Ω^{-1})

Capacitance /1



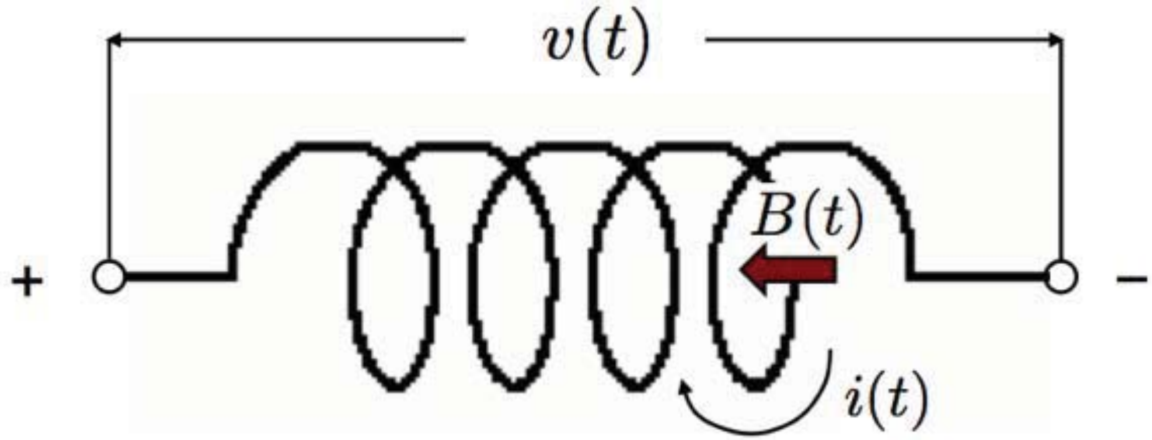
- Since similar charges repel, the potential *v* is necessary to prevent the charges from flowing away from the electrodes (discharge)
- Each change in potential $v(t+\Delta t)=v(t)+\Delta v$ results in change of the energy stored in the capacitor, in the form of charges moving to/away from the electrodes (↔ change in electric field)

Capacitance /2



- Capacitance C: $q(t) = Cv(t) \Rightarrow \frac{\mathrm{d}q(t)}{\mathrm{d}t} \equiv i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t}$
- in Laplace domain: $I(s) = CsV(s) \Rightarrow \frac{V(s)}{I(s)} \equiv Z_C(s) = \frac{1}{Cs}$

Inductance



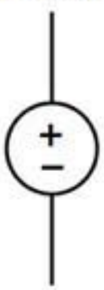
- Current flow *i* around a loop results in magnetic field *B* pointing normal to the loop plane. The magnetic field counteracts changes in current; therefore, to effect a change in current *i*(*t*+Δ*t*)=*i*(*t*)+Δ*i* a potential *v* must be applied (*i.e.*, energy expended)
- Inductance *L*: $v(t) = L\,\frac{\mathrm{d}i(t)}{\mathrm{d}t}$
- in Laplace domain: $V(s) = LsI(s) \Rightarrow \frac{V(s)}{I(s)} \equiv Z_L(s) = Ls$

Summary: passive electrical elements; Sources

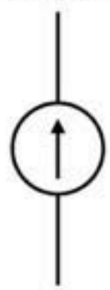
Nise Table 2.3	voltage--current	current--voltage	voltage--charge	Impedance $Z(s) = \frac{V(s)}{I(s)}$.	Conductance $G(s) = \frac{I(s)}{V(s)}$.
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R (Resistance)	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Electrical inputs: voltage source, current source

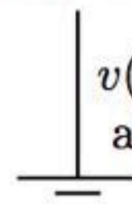


Voltage source:
$v(t)$ independent
of current through.



Current source:
$i(t)$ independent
of voltage across.

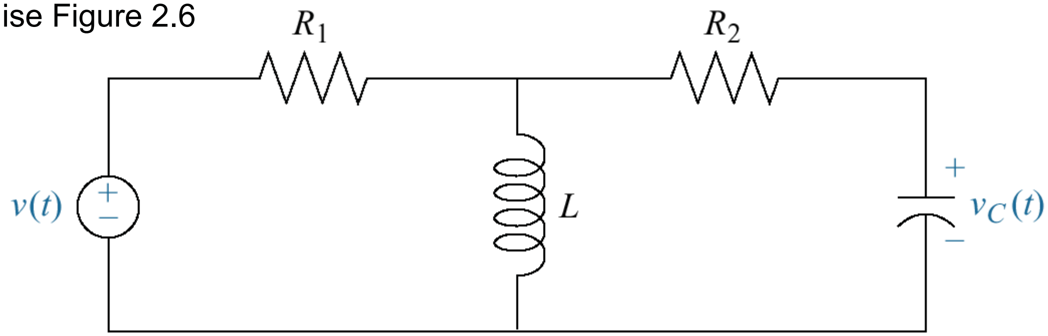
Ground:
potential reference

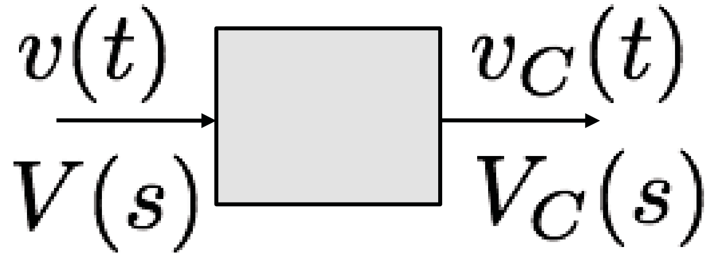


$v(t) = 0$
always

Combining electrical elements: networks

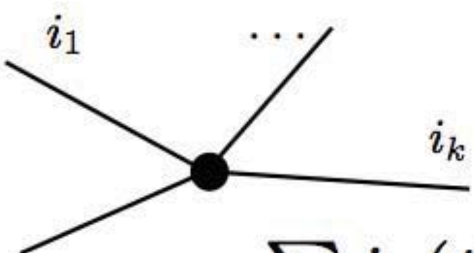
Nise Figure 2.6





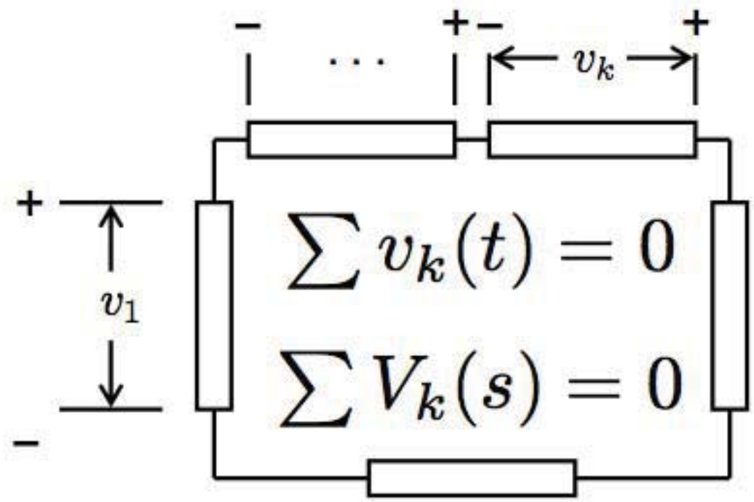
Network analysis relies on two physical principles

- Kirchhoff Current Law (KCL)
  - charge conservation
- Kirchhoff Voltage Law (KVL)
  - energy conservation



$$\sum i_k(t) = 0$$

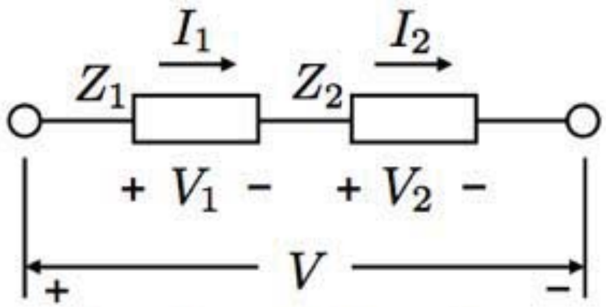
$$\sum I_k(s) = 0$$



$$\sum v_k(t) = 0$$

$$\sum V_k(s) = 0$$

Impedances in series and in parallel



Impedances **in series**

KCL: $I_1 = I_2 \equiv I$.

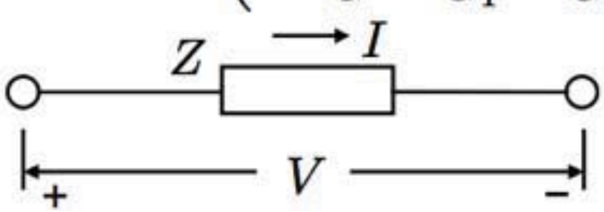
KVL: $V = V_1 + V_2$.

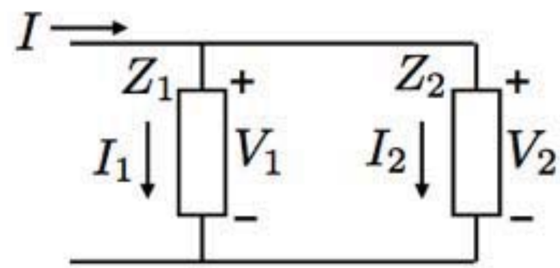
From definition of impedances:

$$Z_1 = \frac{V_1}{I_1}; \qquad Z_2 = \frac{V_2}{I_2}.$$

Therefore, equivalent circuit has

$$Z = Z_1 + Z_2 \left(\Leftrightarrow \frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2}. \right)$$





Impedances **in parallel**

KCL: $I = I_1 + I_2$.

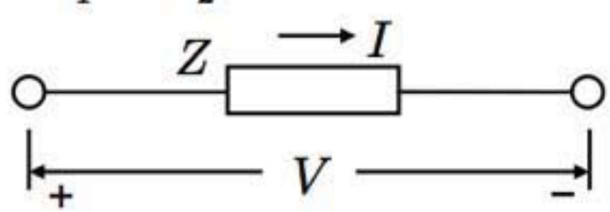
KVL: $V_1 + V_2 \equiv V$.

From definition of impedances:

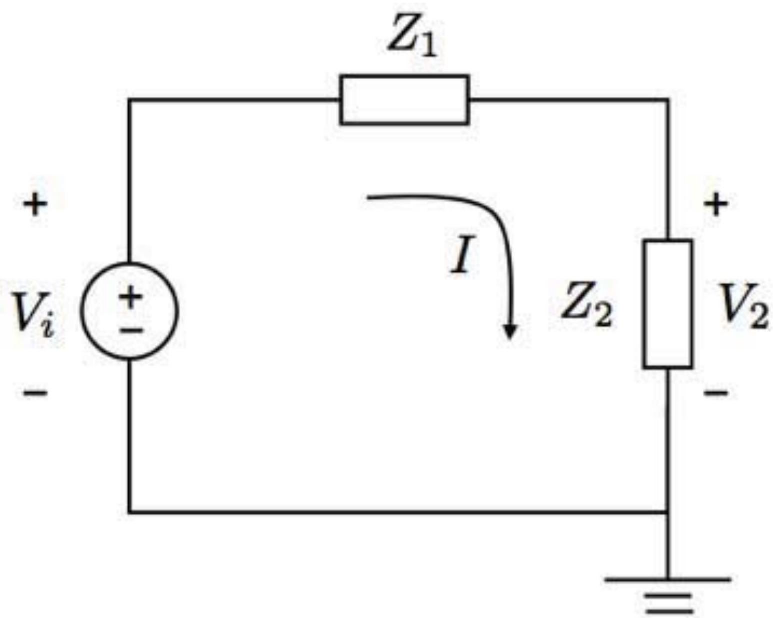
$$Z_1 = \frac{V_1}{I_1}; \qquad Z_2 = \frac{V_2}{I_2}.$$

Therefore, equivalent circuit has

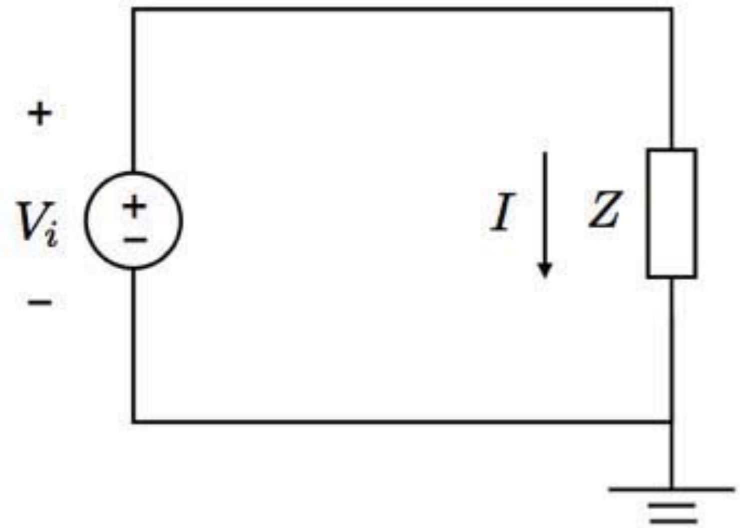
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \left(\Leftrightarrow G = G_1 + G_2. \right)$$



The voltage divider



Equivalent circuit for computing the current I.



Since the two impedances are in series, they combine to an equivalent impedance

$$Z = Z_1 + Z_2.$$

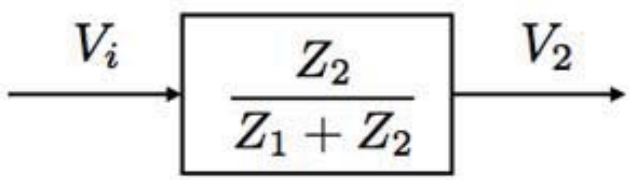
The current flowing through the combined impedance is

$$I = \frac{V}{Z}.$$

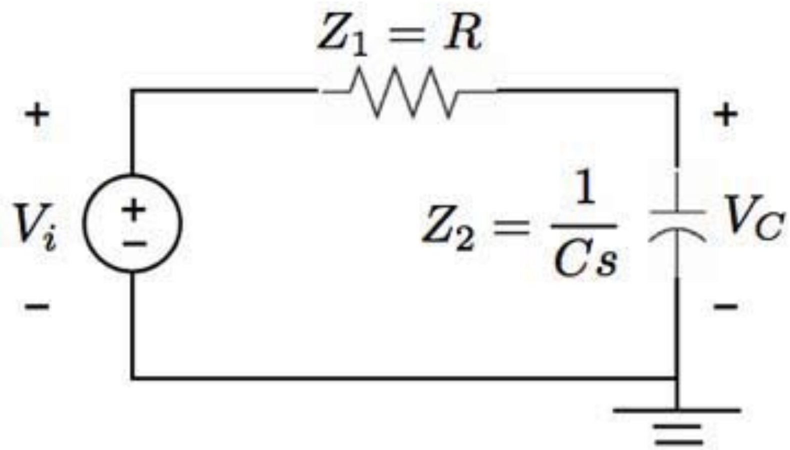
Therefore, the voltage drop across Z_2 is

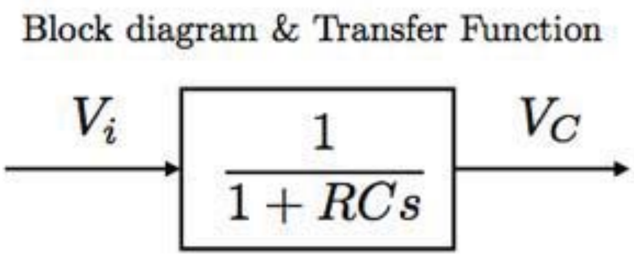
$$V_2 = Z_2 I = Z_2 \frac{V}{Z} \Rightarrow \frac{V_2}{V_i} = \frac{Z_2}{Z_1 + Z_2}.$$

Block diagram & Transfer Function



Example: the *RC* circuit





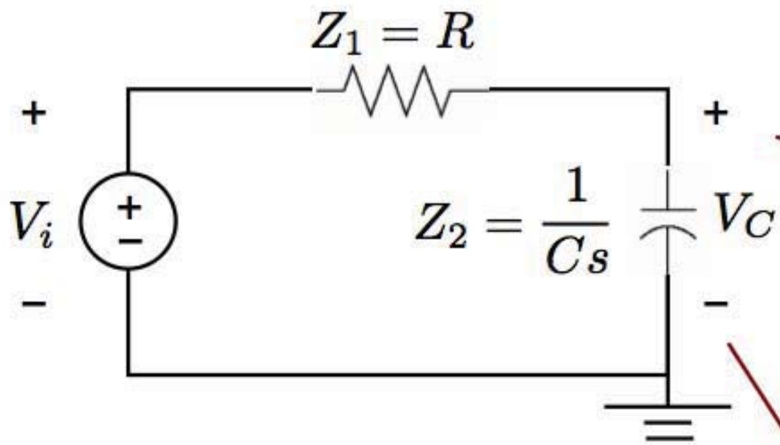
We recognize the voltage divider configuration, with the voltage across the capacitor as output. The transfer function is obtained as

$$\mathrm{TF}(s) = \frac{V_C(s)}{V_i(s)} = \frac{1/Cs}{R+1/Cs} = \frac{1}{1+RCs} = \frac{1}{1+\tau s},$$

where $\tau \equiv RC$. Further, we note the similarity to the transfer function of the rotational mechanical system consisting of a motor, inertia J and viscous friction coefficient b that we saw in Lecture 3. [The transfer function was $1/b(1+\tau s)$, *i.e.* identical within a multiplicative constant, and the time constant τ was defined as J/b.] We can use the analogy to establish properties of the *RC* system without re–deriving them: e.g., the response to a step input $V_i = V_0 u(t)$ (step response) is

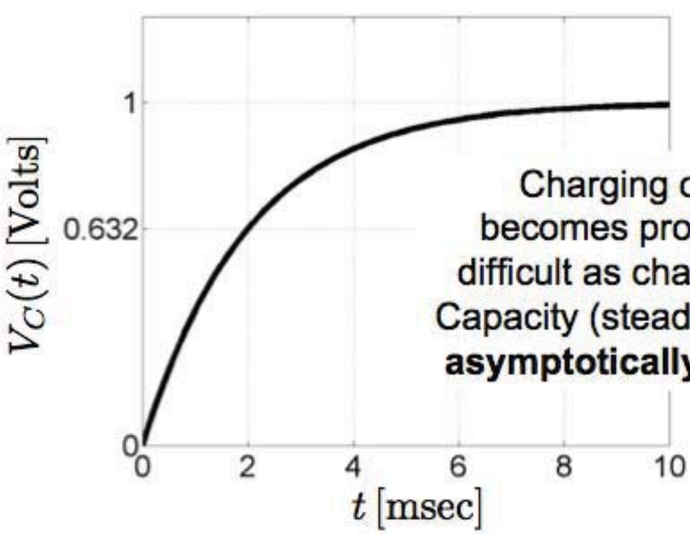
$$V_C(t) = V_0\left(1 - \mathrm{e}^{-t/\tau}\right)u(t), \qquad \text{where now } \tau = RC.$$

Interpretation of the *RC* step response

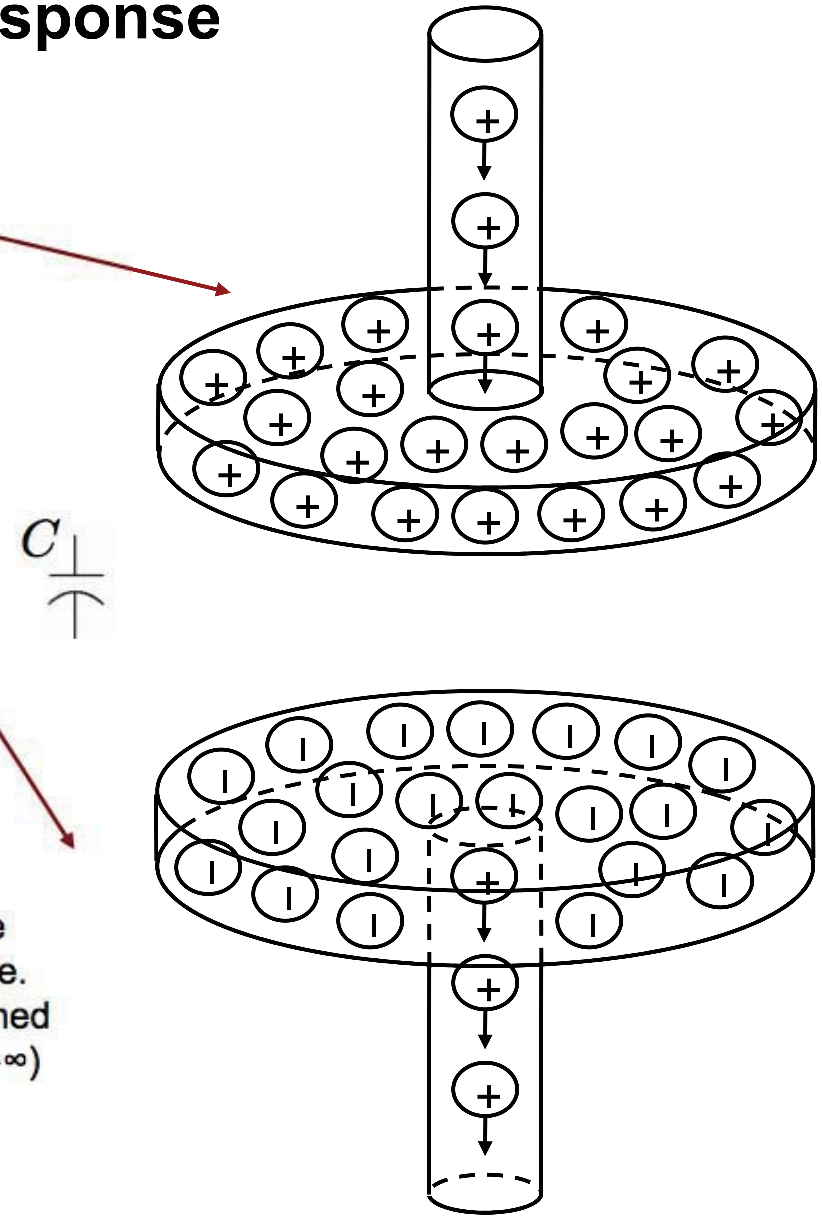


$$V_C(t) = V_0\left(1 - e^{-t/\tau}\right)u(t), \quad \tau = RC.$$

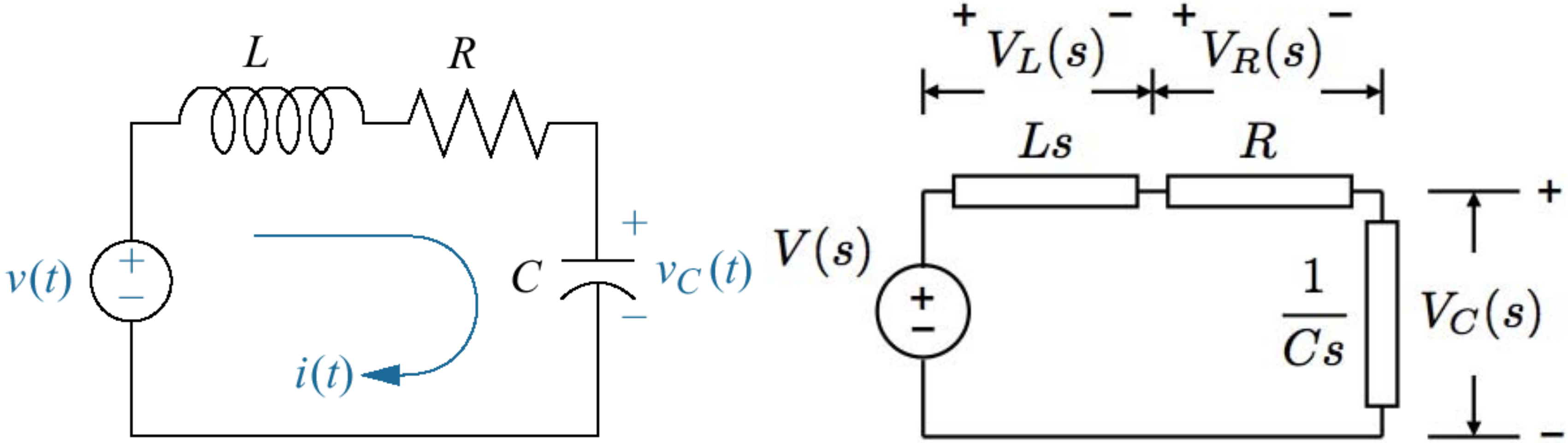
$$V_0 = 1 \text{ Volt} \quad R = 2k\Omega \quad C = 1\mu F$$



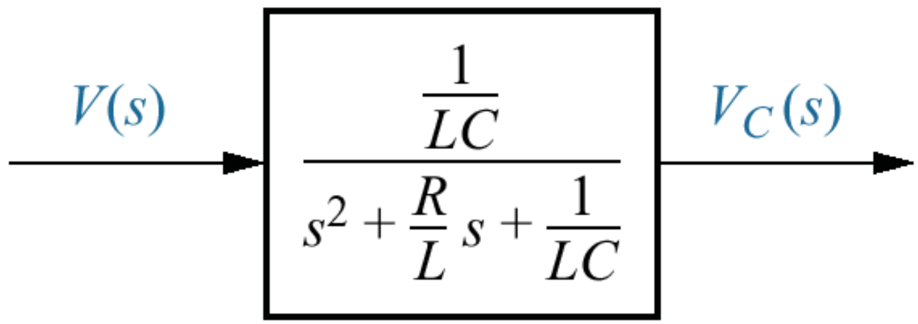
Charging of a capacitor: becomes progressively more difficult as charges accumulate. Capacity (steady-state) is reached **asymptotically** ($V_C \to V_0$ as $t \to \infty$)



Example: RLC circuit with voltage source



Nise Figure 2.3



$$\frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

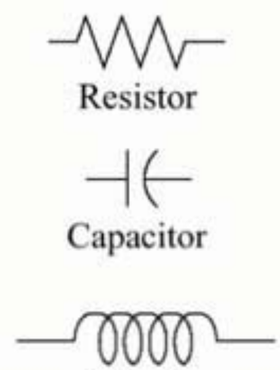
Nise Figure 2.4

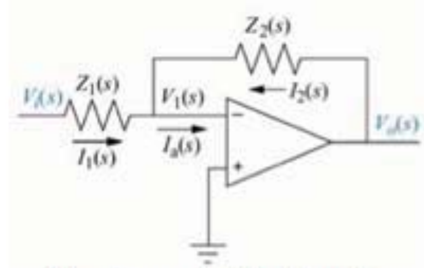
Quick summary of electrical systems

- Electrical dynamical variables and elements

Charge $q(t)$, $Q(s)$.
Current $i(t) = \dot{q}(t)$, $I(s) = sQ(s)$.
Voltage $v(t)$, $V(s) = Z(s)I(s)$.

Resistor $v(t) = Ri(t)$, $Z_R(s) = R$.
Capacitor $i(t) = C\dot{v}(t)$, $Z_C(s) = 1/Cs$.
Inductor $v(t) = L\mathrm{d}i(t)/\mathrm{d}t$, $Z_L(s) = Ls$.

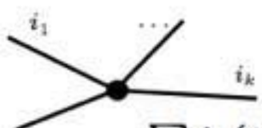




Op–Amp in feedback configuration:

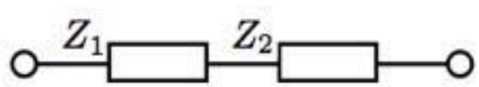
$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}.$$

- Electrical networks



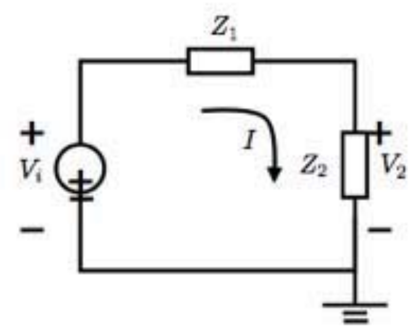
KCL

$$\sum i_k(t) = 0$$
$$\sum I_k(s) = 0$$

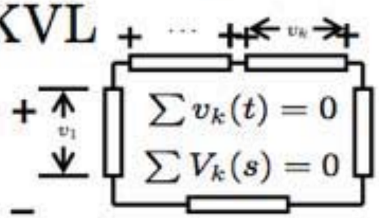


Impedances in series

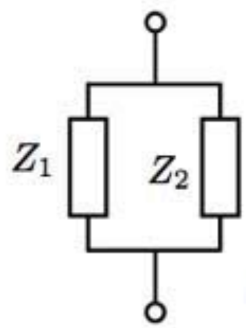
$$Z = Z_1 + Z_2 \quad \frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2}.$$



KVL

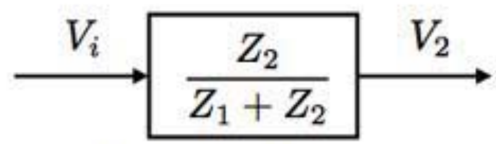


$$\sum v_k(t) = 0$$
$$\sum V_k(s) = 0$$



Impedances in parallel

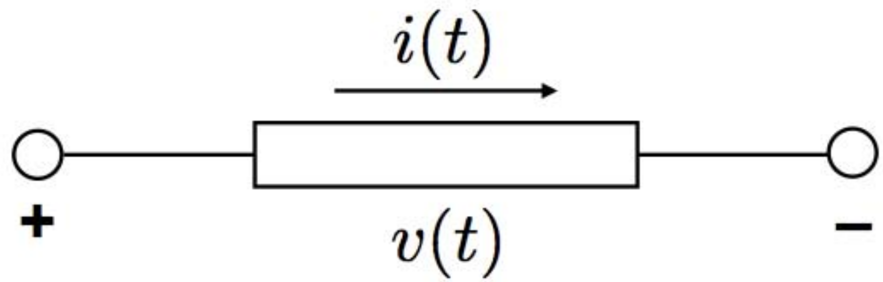
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \quad G = G_1 + G_2.$$



$$\frac{Z_2}{Z_1 + Z_2}$$

Voltage divider

Power dissipation in electrical systems



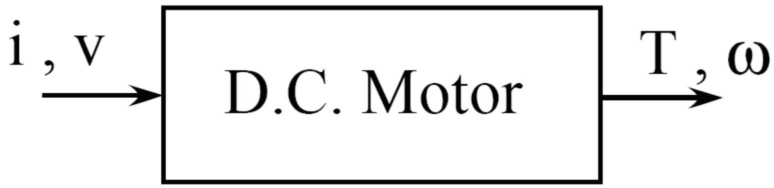
Instantaneous power dissipation

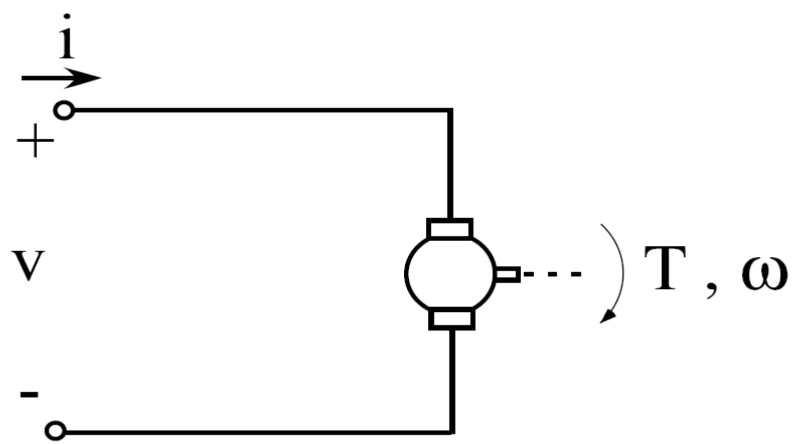
$$P(t) = i(t) \cdot v(t).$$

Unit of power: 1 Watt=1 A · 1 V.

<u>NOTE:</u> $P(s) \neq I(s) \cdot V(s)$. **Why?**

DC Motor as a system





$$P_{in} = P_{out}$$

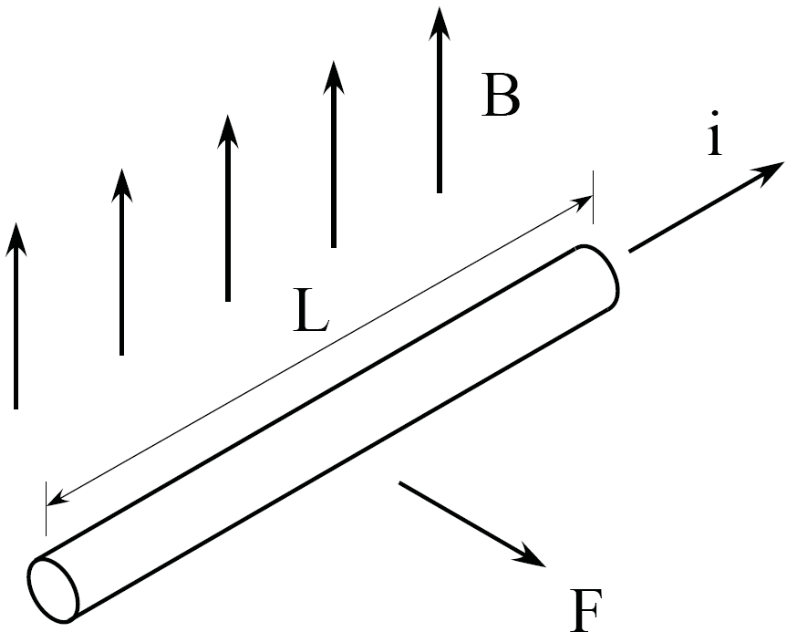
$$i(t) * v(t) = T(t) * \omega(t)$$

Transducer:

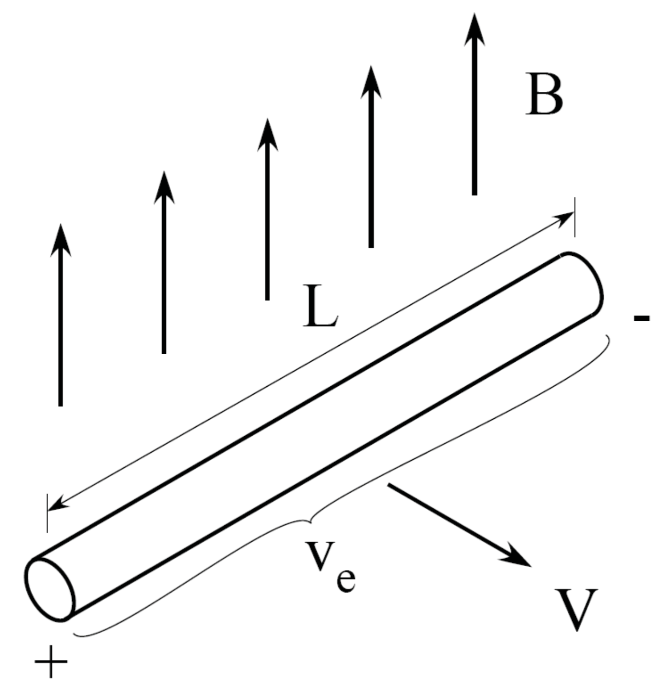
Converts energy from one domain (electrical)
to another (mechanical)

Physical laws applicable to the DC motor

Lorentz law:
magnetic field applies force to a current (Lorentz force)



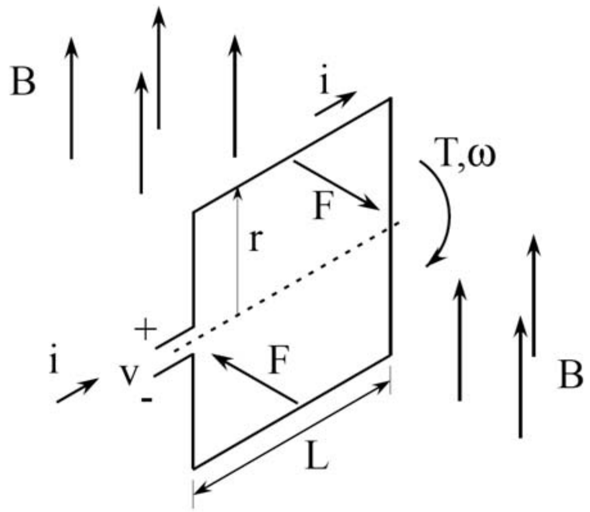
Faraday law:
moving in a magnetic field results in potential (back EMF)

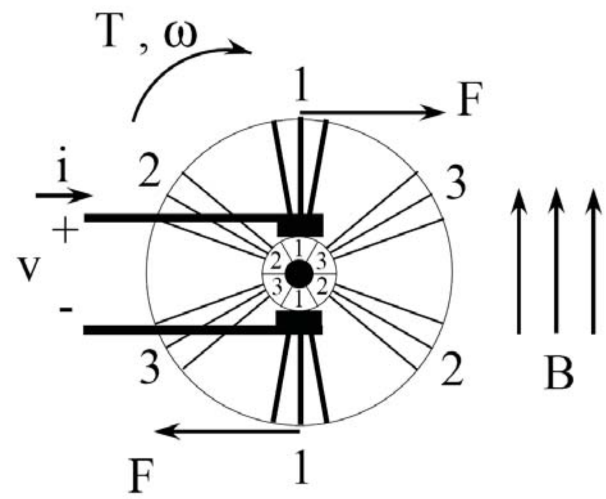


$$F = (\mathbf{i} \times \mathbf{B}) \cdot l = iBl \quad (\mathbf{i} \perp \mathbf{B})$$

$$v_e = \mathbf{V} \times \mathbf{B} \cdot l = VBl \quad (\mathbf{V} \perp \mathbf{B})$$

DC motor: principle and simplified equations of motion





multiple windings *N*:
continuity of torque

$$T = 2Fr = 2(iBNl)r \quad \text{(Lorentz law)}$$

$$v_e = 2VBNl = 2(\omega r)BNl \quad \text{(Faraday law)}$$

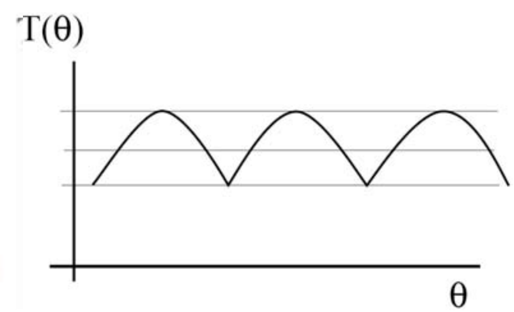
or

$$T = K_m i$$

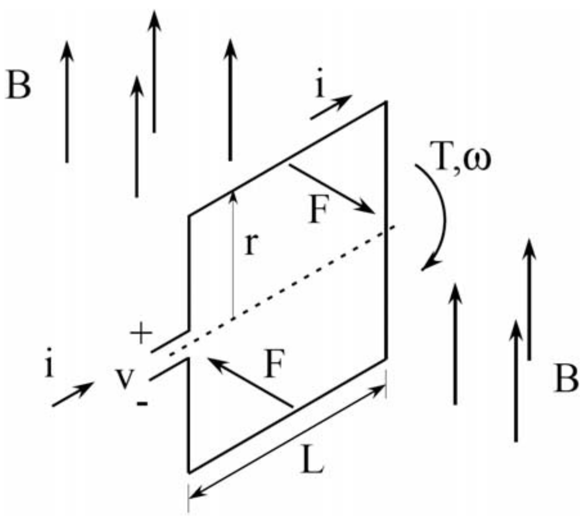
$$v_e = K_v \omega$$

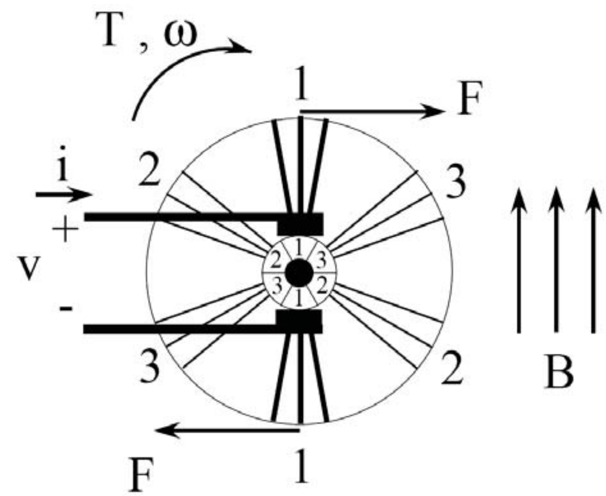
where

- $K_m \equiv 2BNlr$ torque constant
- $K_v \equiv 2BNlr$ back-emf constant

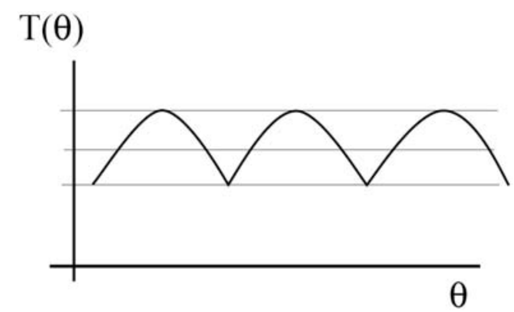


DC motor: equations of motion in matrix form





multiple windings *N*:
continuity of torque

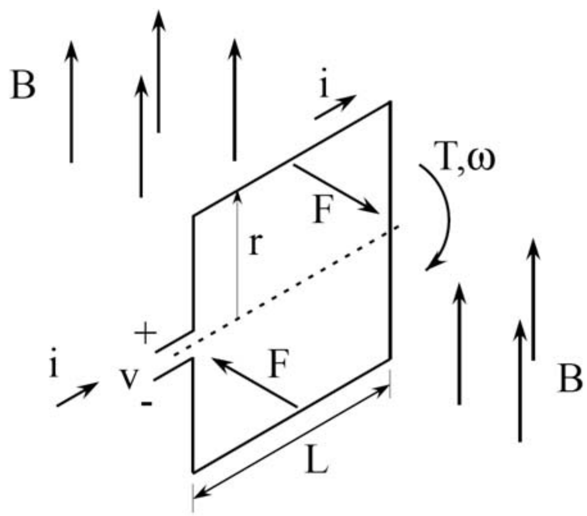


$$\begin{bmatrix} v_e \\ i \end{bmatrix} = \begin{bmatrix} 2BNlr & 0 \\ 0 & \frac{1}{2BNlr} \end{bmatrix} \begin{bmatrix} \omega \\ T \end{bmatrix}$$

or

$$\begin{bmatrix} v_e \\ i \end{bmatrix} = \begin{bmatrix} K_v & 0 \\ 0 & \frac{1}{K_m} \end{bmatrix} \begin{bmatrix} \omega \\ T \end{bmatrix}$$

DC motor: why is $K_m = K_v$?



$$P_{in} = P_{out}$$

$$i(t) * v(t) = T(t) * \omega(t)$$

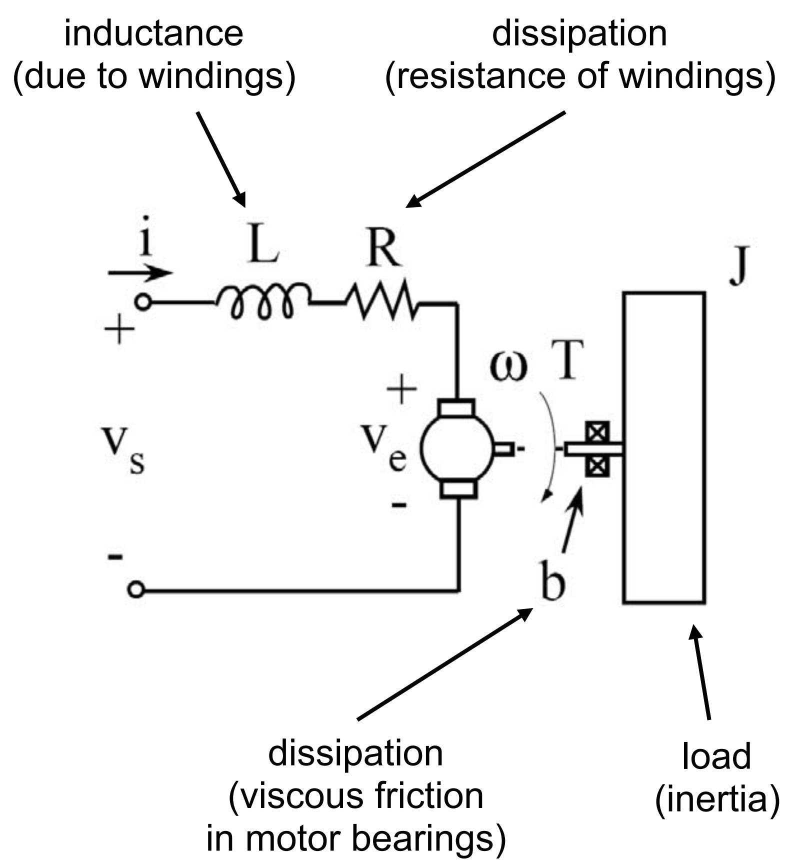
$$P_{in} = P_{out} \qquad \text{(power conservation)}$$

$$\Rightarrow i v_e = T\omega$$

$$\Rightarrow K_v i\omega = K_m i\omega$$

$$\Rightarrow K_v = K_m \qquad \text{QED.}$$

DC motor with mechanical load and realistic electrical properties (*R*, *L*)



Equation of motion – Electrical

$$\text{KCL:} \quad v_s - v_L - v_R - v_e = 0$$

$$\Rightarrow v_s - L\frac{di}{dt} - Ri - K_v\omega = 0$$

Equation of motion – Mechanical

$$\text{Torque Balance:} \quad T = T_b + T_J$$

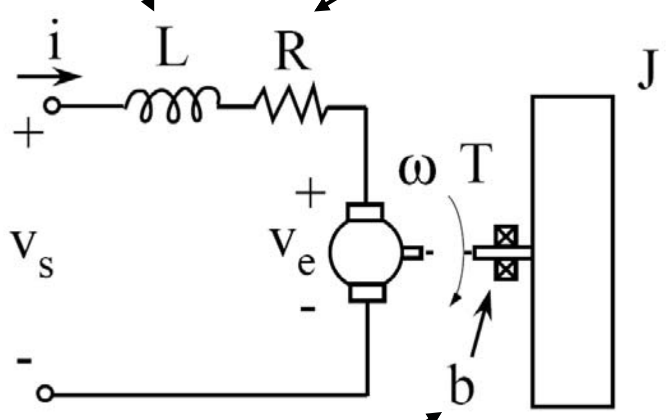
$$\Rightarrow K_m i - b\omega = J\frac{d\omega}{dt}$$

Combined equations of motion

$$L\frac{di}{dt} + Ri + K_v\omega = v_s$$

$$J\frac{d\omega}{dt} + b\omega = K_m i$$

DC motor with mechanical load and realistic electrical properties (*R*, *L*)



Equation of motion – Electrical

$$\text{KCL:} \quad V_s(s) - V_L(s) - V_R(s) - V_e(s) = 0$$

$$V_s(s) - LsI(s) - RI(s) - K_v\Omega(s) = 0$$

Equation of motion – Mechanical

$$\text{Torque Balance:} \quad T(s) = T_b(s) + T_J(s)$$

$$K_m I(s) - b\Omega(s) = Js\Omega(s)$$

Combined equations of motion

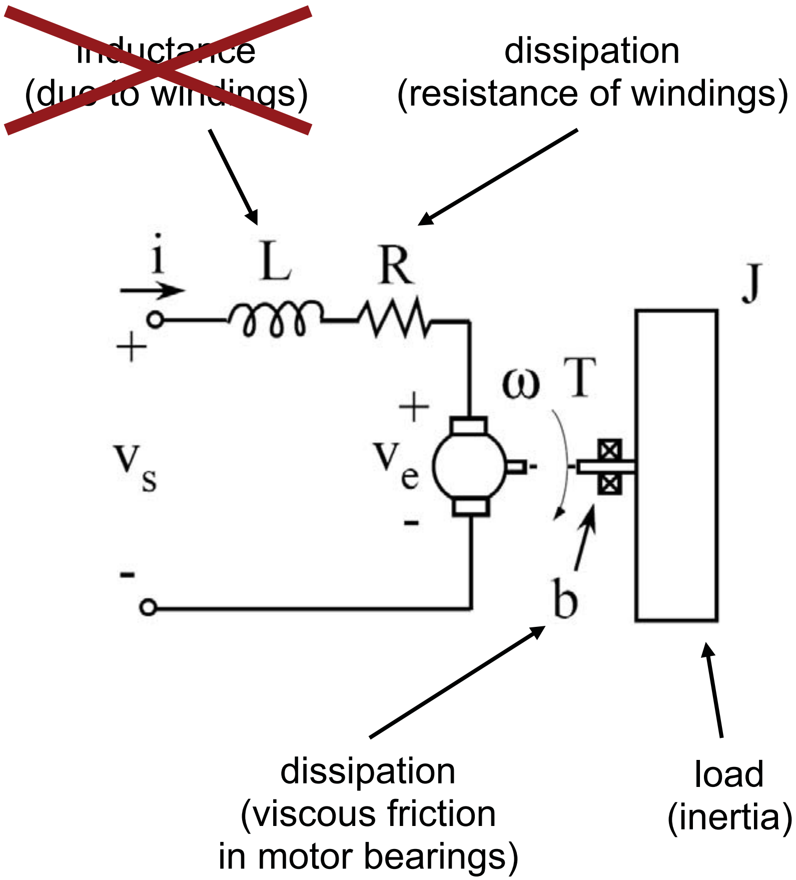
$$LsI(s) + RI(s) + K_v\Omega(s) = V_s(s)$$

$$Js\Omega(s) + b\Omega(s) = K_m I(s)$$

$$\Rightarrow \left[(Ls+R)\left(\frac{Js+b}{K_m}\right) + K_v\right]\Omega(s) = V_s(s)$$

$$\Rightarrow \left[\frac{LJ}{R}s^2 + \left(\frac{Lb}{R} + J\right)s + \left(b + \frac{K_m K_v}{R}\right)\right]\Omega(s) = \frac{K_m}{R}V_s(s)$$

DC motor with mechanical load and realistic electrical properties (R, ~~L~~)



Neglecting the impedance

$$L \approx 0$$

$$\Rightarrow \left[Js + \left(b + \frac{K_m K_v}{R} \right) \right] \Omega(s) = \frac{K_m}{R} V_s(s)$$

This is our familiar 1st–order system!

If we are given step input $v_s(t) = V_0 u(t)$
$\Rightarrow$ we already know the step response

$$\omega(t) = \frac{K_m}{R} V_0 \left(1 - \mathrm{e}^{-t/\tau} \right) u(t),$$

where now the time constant is

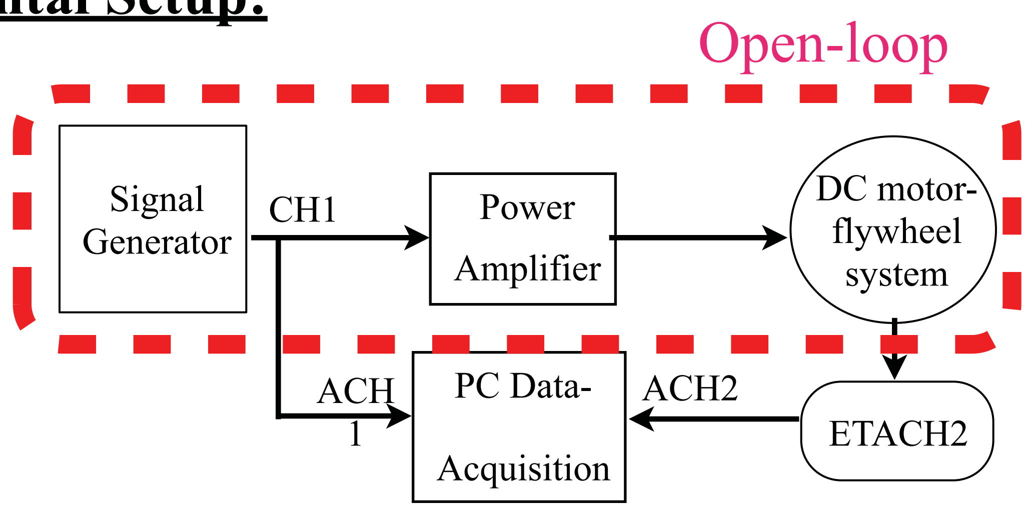
$$\tau = \frac{J}{\left(b + \dfrac{K_m K_v}{R} \right)}.$$

Lab 06: Running the flywheel with DC motor open loop

- Observe motor behavior under different driving voltages.
- Examine transient response of a DC motor

Important: always be ready to turn off the break of power amplifier when motor is spinning too fast!!

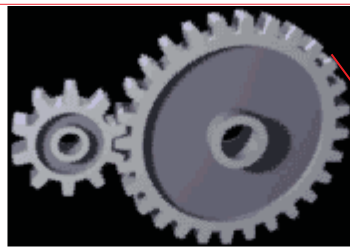
Experimental Setup:



Procedure

- Make sure all devices are powered off; connect function generator, power amplifier, flywheel system and PC data acquisition system as shown in the previous slide.
- Add one magnet to the flywheel damper, open Chart Recorder to record ACH1 and ACH2; turn on function generator, power amplifier and start experiment.
- Obtain system response for a ramp function with Freq: 0.2 Hz, Amp: 0.5 V, offset: 0 V. Repeat experiment using a sine function with same parameters. Record your response. Referring to materials we learned from last lecture, comment on the behavior of DC motor- flywheel system.
- Use a DC signal with 0.2 V offset. Start experiment and record DC motor transient response data. Convert voltage signal to **motor speed** (you will need to make use of gear ratio). Generate appropriate plots of **motor speed** & amplified current V.S time. Compute mechanical power of the DC motor.
- Set your function generator to generate a square function(SQUA), set frequency to 0.04 Hz, amplitude to 0.200V and offset to 0.100V. Collect a full period of flywheel response and function generator signal. (You can take a screen shots of the plot in Chart Recorder)

Some Hints…



Gear 1 Gear 2

F

Gear Ratio: $\frac{n_1}{n_2} = \frac{r_1}{r_2} = \frac{\Omega_2}{\Omega_1} = \frac{T_1}{T_2} = \frac{F_1 r_1}{F_2^T r_2}$

$$\frac{n_1}{n_2} = \frac{44}{180}$$

Unit Conversion:

$$rpm = \frac{2\pi}{60}(rad/s)$$

$$N = \frac{kg \times m}{s^2}$$

$$V(voltage) = \frac{kg \times m^2}{s^3 \times A}$$

Power Conservation:

$$P_{mechanical} = P_{electrical}$$

$$K_a = 2.0 \text{ A/V}, \quad K_m = 0.0292 \text{ N-m/A}$$

$$\begin{aligned} P_{mechanical}(t) &= T(t) \times \Omega(t) \\ &= K_m \times i(t) \times \Omega(t) \end{aligned}$$

Lab assignment p.1

- Comment on how today's experiments involving step input are *interpreted* differently than we did in Lab 05.

- When the DC motor is driven by a step function, how many poles/ zeros do we need to consider, and where are they? How do the magnets ("eddy brakes") influence their locations?

Lab assignment p.2

- When the DC motor is driven by a ramp function, how many poles/zeros do we need to consider, and where are they? How do the magnets ("eddy brakes") influence their locations?

- Comment on the qualitative and quantitative agreement of your derivations with experiment. Attach sheets with your experimental results.

2.04A Systems and Controls
Spring 2013