1.138J/2.062J/18.376J Wave Propagation

Take-Home Exam

This is a closed-book exam. You may use only your own class notes, problem sets and the lecture notes posted on the 1.138J/2.062J/18.376J website. You are not allowed to discuss this exam with anyone else.

Problem 1 (10 points)

A sandwich construction consists of three elastic layers as sketched below.

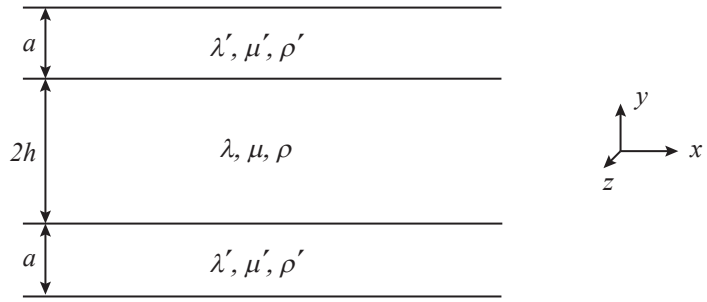
(a) Study the propagation of SH waves in this structure by considering displacement solutions of the form

$$u = v = 0, \qquad w = f(y)e^{ik(x-ct)}$$

together with the appropriate boundary/interface conditions.

(b) Obtain the corresponding dispersion relation

Hint: Exploiting symmetry, simplify the algebra by dividing the propagation modes into symmetric and anti-symmetric



Problem 2 (10 points)

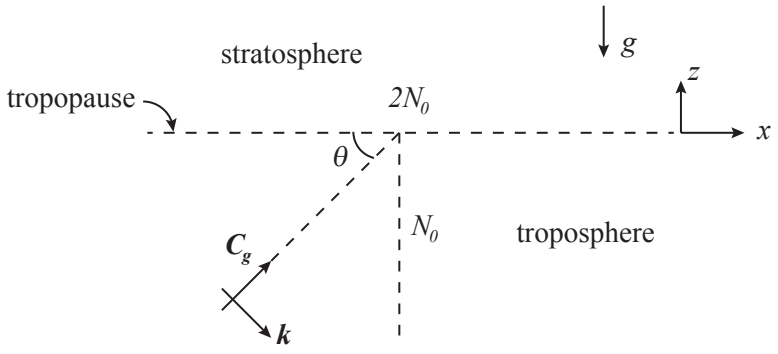
In the atmosphere, the buoyancy frequency N varies with altitude. Specifically, N experiences a rapid increase, by roughly a factor of 2, in the so-called tropopause – the region between the troposphere and stratosphere, typically found 10–15 km from the ground.

A simple two-layer model of the troposphere–stratosphere transition is sketched below. The two semi-infinite layers have constant buoyancy frequency, but in the troposphere ($z < 0$) $N = N_0$ while in the stratosphere ($z > 0$) $N = 2N_0$. The tropopause is idealized as a sharp interface ($z = 0$), where the density is continuous but the buoyancy frequency experiences a jump. The Boussinesq approximation is assumed to be valid throughout.

Suppose a harmonic plane wave

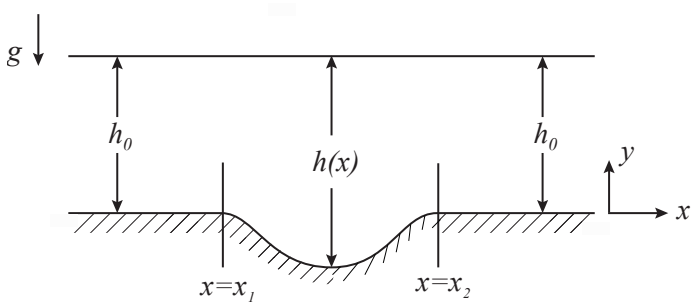
$$w_{inc} = A_{inc} \exp\{ik(x \sin\theta - z \cos\theta) - i\omega t\} + cc,$$

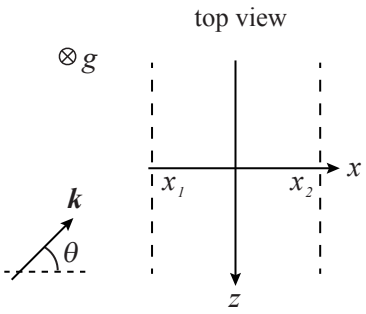
where w denotes the vertical velocity component and $\omega = N_0 \sin\theta$ the wave frequency, is incident on the tropopause ($z = 0$) from the troposphere ($z < 0$), as sketched below. Determine the reflected wave w_{refl} in the troposphere and the transmitted wave w_{trans} in the stratosphere ($z > 0$).



Problem 3 (10 points)

A submarine trough connects two sides of equal water depth. A harmonic surface wave approaches the trough from $x \to -\infty$ obliquely at an angle of incidence θ, as sketched below. Use ray theory to discuss the refraction of this wave by the trough as a function of the angle θ.





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