Lecture 3
The Concept of Stress, Generalized Stresses and Equilibrium

Problem 3-1: Cauchy's Stress Theorem

Cauchy's stress theorem states that in a stress tensor field there is a traction vector ***t*** that linearly depends on the outward unit normal ***n***:

$$\boldsymbol{t} = \boldsymbol{\sigma n}$$

a. Express Cauchy's stress theorem in index form.

b. Suppose the state of stress at a point in *x, y, z* coordinate system is given by the matrix below. Determine the normal stress σ_n and the shear stress τ on the surface defined by .

Problem 3-2: Three invariants of a stress tensor

Suppose the state of stress at a point in a *x, y, z* coordinate system is given by

$$\begin{bmatrix} 100 & 1 & 180 \\ 0 & 20 & 0 \\ 180 & 0 & 20 \end{bmatrix}$$

a. Calculate the three invariants of this stress tensor.

b. Determine the three principal stresses of this stress tensor.

Problem 3-4: Transformation Matrix

Suppose the state of stress at a point relative to a *x, y, z* coordinate system is given by:

$$\begin{bmatrix} 15 & -10 \\ -10 & -5 \end{bmatrix}$$

Try to find a new coordinate system *(x', y')* that corresponds to the principal directions of the stress tensor.

a. Find the principal stresses.

b. Determine the transformation matrix $[L]$.

c. Verify $[\sigma]' = [L]^T[\sigma][L]$.

Problem 3-5: Beam Equilibrium

Derive the equation of force and moment equilibrium of a beam using the equilibrium of an infinitesimal beam element of the length dx.

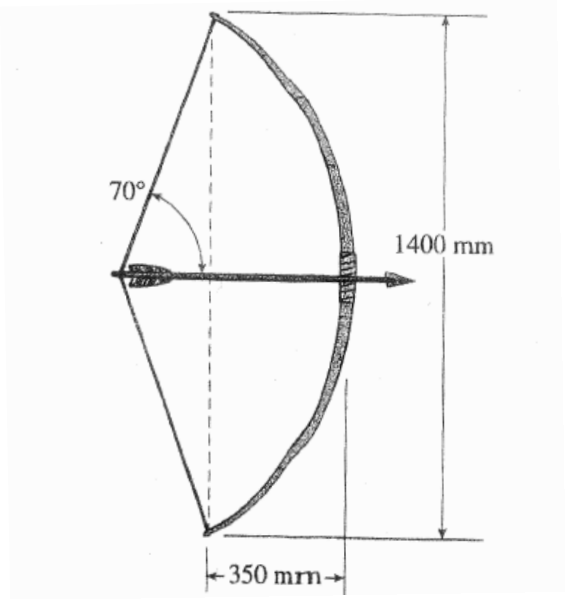
Problem 3-6: Moderately large deflections in beams

Explain which of the three equilibrium equations below is affected by the finite rotations in the theory of the moderately large deflections of beam.

1. Moment equilibrium
2. Vertical force equilibrium
3. Axial force equilibrium

Problem 3-7:

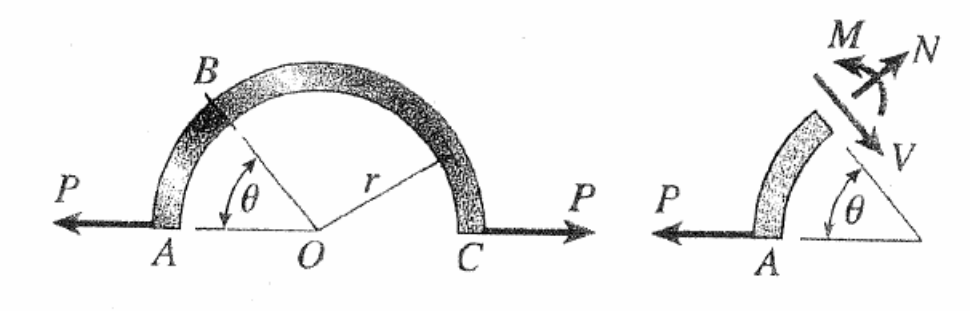
At full draw, and archer applies a pull of 150N to the bowstring of the bow shown in the figure. Determine the bending moment at the midpoint of the bow.



Problem 3-8:

A curved bar ABC is subjected to loads in the form of two equal and opposite force P, as shown in the figure. The axis of the bar forms a semicircle of radius r.

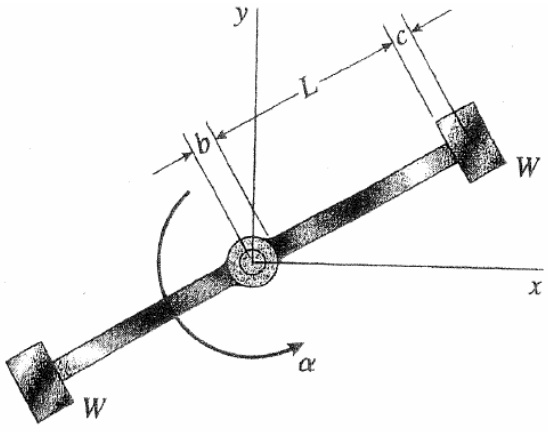
Determine the axial force N, shear force V, and bending moment M acting at a cross section defined by the angle θ



Problem 3-9:

The centrifuge shown in the figure rotates in a horizontal plane (the xy plane) on a smooth surface about the z axis (which is vertical) with an angular acceleration α . Each of the two arms has weight w per unit length and supports a weight $W = 2.5wL$ at its end.

Derive formulas for the maximum shear force and maximum bending moment in the arms, assuming $b = L/9$ and $c = L/10$

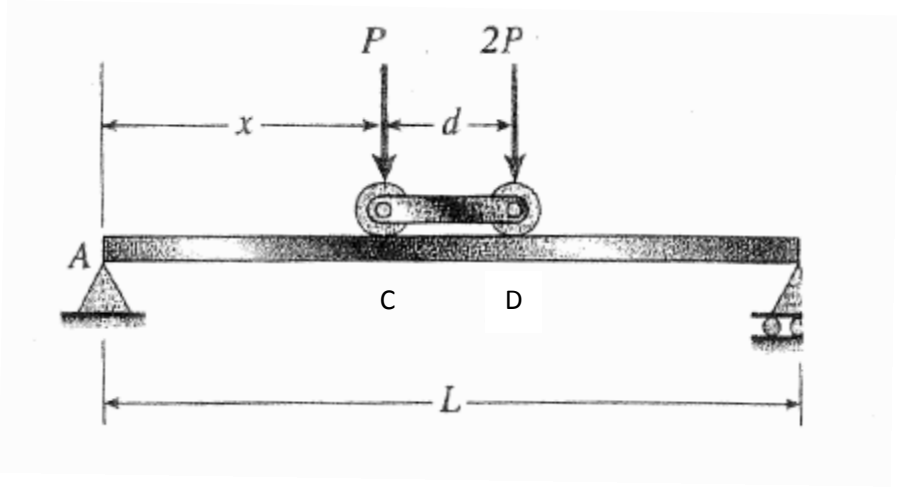


Problem 3-10:

A simple beam AB supports two connective loads P and 2P that are distance *d* apart (see figure). The wheels may be placed at any distance x from the left support of the beam.

(a) Determine the distance x that will produce maximum shear force in the beam, and also determine the maximum shear force V_{max}

(b) Determine the distance that will produce the maximum bending moment, and also draw the corresponding bending moment diagram. (Assume P=10kN, d=2.4m, and L=12m)



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