

Quiz 2**October 30, 2013****Problem 1 Solutions:**

A long prismatic square tube, consisting of 4 plates, is subjected to axial compression. The geometry is defined by the wall thickness t and the width b . Explain the entire process of elastic-plastic response of the tube that will include answers to the following questions:

a) *Justify the boundary conditions for each individual plate along the edges*

Simply supported, the angle between the edges of the plate is fixed, but if the buckling can occur in opposite directions they can rotate, therefore we can consider the boundary conditions simply supported.

b) *Determine the buckling load and stress*

$$N_c = k_c \frac{\pi^2 D}{b^2} \text{ where } D = \frac{Eh^3}{12(1-\nu)} \text{ (Eq. 11.15)}$$

Assume that $a > b$; therefore $k_c = 4$

$$\text{Buckling load for the tube is } N = 16 \frac{\pi^2 D}{b^2}$$

$$\text{Buckling stress for the tube is } \sigma = 4 \frac{\pi^2 D}{tb^3}$$

c) *Find the pre-buckling and post-buckling stiffness of the column*

$$K_{pre} = 4Eh$$

Eq. 11-44 (additional factor 4 because the force is divided over all 4 plates)

$$K_{post} = \frac{13}{25} (1 - \nu^2) C \text{ where } C = \frac{Eh}{(1 - \nu^2)} = 2.08Eh = 0.52K_{pre}$$

Eq. 11-48

d) *State the expression for the ultimate stress and explain where it comes from*

$$\sigma_{ult} = \sqrt{\sigma_{cr} \sigma_y}$$

$$\sigma_{cr} = \left(\frac{1.9}{\beta} \right)^2 \sigma_y$$

Substitute expressions

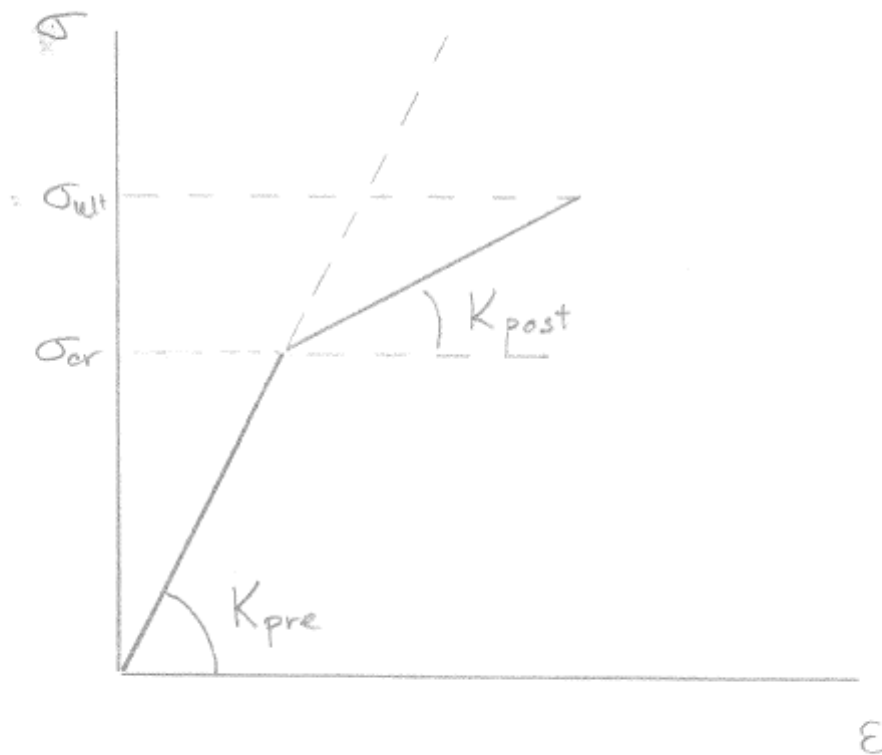
$$\sigma_{ult} = \left(\frac{1.9}{\beta} \right) \sigma_y$$

$$\beta = \frac{b}{h} \sqrt{\frac{E}{\sigma_y}}$$

$$\sigma_{ult} = \frac{1.9t}{b} \sqrt{E\sigma_y}$$

e) Reconstruct the entire load-displacement curve for the column including pre-buckling phase, buckling point, post-buckling response, and ultimate load.

Assume that the yield stress is greater than the ultimate stress.



Problem 2:

The problem is concerned with the tallest column covered on pages 10-1 to 10-5 of the lecture notes.

Consider the following: Google and Microsoft are competing to erect the tallest column. Microsoft would like to use traditional steel, while a more innovative Google would like to build it from aluminum. Predict who will win the contest and explain why. Use the following material properties:

Material	Mass Density (g/cm ³)	Young's Modulus (GPa)
Steel	7.8	210
Aluminum	2.7	69

From Eq. 10-12:

$$l_c^3 = 7.837 \frac{EI}{q}$$

where q is the mass density of the material.

We assume that the I is equal in both columns; then the length is proportional to the cube root of the ratio of the stiffness and the mass density.

$$\text{Aluminum: } \frac{E}{q} = \frac{69}{2.7} = 25.5$$

$$\text{Steel: } \frac{E}{q} = \frac{210}{7.8} = 26.9$$

Given this information, steel (Microsoft) would be able to construct a taller tower; however, let us examine the magnitude of the difference. Consider the cube root of the ratios and find their percentage difference.

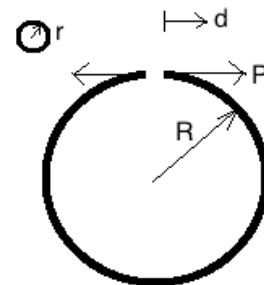
$$\frac{\sqrt[3]{r_{st}} - \sqrt[3]{r_{al}}}{\sqrt[3]{r_{al}}} * 100\% = 1.75\%$$

This difference is minimal and an interesting outcome for the problem.

Problem 3:

Consider a key ring with a circular cross-section. It has a radius R and cross sectional radius r shown in the sketch below. Derive an expression for the opening displacement as a function of the load P , geometry (R and r), and Young's modulus, E , using energy methods.

Note: $\int_0^\pi \cos^2 x dx = \frac{\pi}{2}$ and $I_{circle} = \frac{\pi r^4}{4}$



Using Castigliano's method:

$$\frac{\partial U}{\partial P} = w_0$$

$$U = \frac{1}{2EI} \int_0^\pi M^2 dR\theta$$

$$M(\theta) = PR(1 - \cos\theta)$$

Substitute into U

$$U = \frac{P^2 R^3}{2EI} \left(\theta - 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{2} \right)$$

The expression within the limits of the integral becomes

$$U = \frac{3\pi P^2 R^3}{4EI}$$

Take the partial derivative with respect to P and include the expression of I for the cross section

$$U = \frac{3P^2 R^3}{Er^4}$$

$$\frac{\partial U}{\partial P} = w_0 = \frac{6PR^3}{Er^4}$$

Extra Credit:

Design a ring so that you can insert a key of thickness 2mm using the force of your own fingers. Clearly state your assumptions.

Assume the coefficient of static friction between the ring and your fingers is 0.75 and you can apply a force of 10N with your fingers and the pulling apart force is greater than 10N.

Using the answer from Problem 3, we can solve for the ratio of R to r in order to meet the displacement criteria:

$$R = \sqrt[3]{\frac{w_0 Er^4}{6P}}$$

Assume the ring is made from steel with $E = 210 \text{ GPa}$

Next, solve for the maximum P we can apply.

$$P_{max} = \mu N = 0.75 * 10N = 7.5N$$

Substitute known values into the expression above; choose an arbitrary cross section $r = 1\text{mm}$, and the radius of the ring, R , is approximately 21mm

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2.080J / 1.573J Structural Mechanics
Fall 2013

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