2.086 Numerical Computation for Mechanical Engineers

MINI-QUIZ 7 — Fall 2014

You may refer to the textbook, lecture notes, Matlab® tutorials, and other class materials as well as your own notes and scripts.

You may use a calculator (for simple arithmetic operations and function evaluations). However, laptops, tablets, and smartphones are not permitted.

You have 30 minutes of recitation to complete the mini quiz. When you are finished, you can hand in your quiz and start working on your assignment.

NAME ________________________________

There are a total of 100 points: four questions, each worth 25 points. Question 4 has two parts, worth 13 and 12 points, respectively.

All questions are multiple choice; ***circle one and only one answer***. Make sure to fully erase or indicate "Retracted" on any other circles not associated with your single final answer.

We provide two blank pages at the end of the quiz which you may use for any derivations, but note that we will *only grade your multiple choice selections*.

This (same) quiz will be administered in all recitation sections. *You may not discuss this quiz with other people until the graded quizzes are returned to the class.*

Question 1 (25 points). Given a real $n \times n$ matrix A that is symmetric positive definite, which statement is **not** true?

(a) All eigenvalues of A are positive

(b) $A = A^T$

(c) For any real $n \times 1$ vector w, the product $f = Aw$ produces a real $n \times 1$ vector f whose elements $f_i,\ i = 1, ..., n$ are all positive

(d) For any real $n \times 1$ vector f, the system $Au = f$ has a unique solution $u = A^{-1}f$

Question 2 (25 points). Let A be an $n \times n$ real matrix and u, v, w and b be real, non-zero vectors of size $n \times 1$. Here we consider $n \geq 2$. Which of the following statements is correct for any A?

(a) If the system $Au = v$ has a solution, u, then the system $Aw = b$ also has a solution, w.

(b) If the linear system $Au = v$ has a unique solution, u, then A has positive eigenvalues.

(c) If A has at least one non-zero eigenvalue, then the system $Au = v$ has a unique solution, u.

(d) If the system $Au = v$ has a unique solution, u, then the system $Aw = b$ also has a unique solution, w.

Question 3 (25 points). Consider the 2×2 system of equations $Au = f$ for

$$A = \begin{pmatrix} 6 & 2 \\ 9 & 3 \end{pmatrix}$$

If $f = (1 \ \ 1)^T$ then

(a) A unique solution, u, exists

(b) No solution, u, exists

(c) Two (different) solutions for u exist

(d) More than two (different) solutions for u exist

Question 4 (25 points). Figure 1 shows a triple pendulum made of 3 Wilberforce springs. Wilberforce springs are special flexural elements that couple extension and rotation and as a result, tend to twist when extended/contracted and tend to extend/contract when twisted.

The **equilibrium equations** for the system shown in the figure can be written as follows

$$
\begin{aligned}
2kx_1 - kx_2 + 2\epsilon\theta_1 - \epsilon\theta_2 &= 0 \\
2\epsilon x_1 - \epsilon x_2 + 2\delta\theta_1 - \delta\theta_2 &= 0 \\
-kx_1 + 2kx_2 - kx_3 - \epsilon\theta_1 + 2\epsilon\theta_2 - \epsilon\theta_3 &= 0 \\
-\epsilon x_1 + 2\epsilon x_2 - \epsilon x_3 - \delta\theta_1 + 2\delta\theta_2 - \delta\theta_3 &= 0 \\
-kx_2 + kx_3 - \epsilon\theta_2 + \epsilon\theta_3 &= F \\
-\epsilon x_2 + \epsilon x_3 - \delta\theta_2 + \delta\theta_3 &= T
\end{aligned} \tag{1}
$$

where x_1, x_2 and x_3 denote the locations of masses 1, 2, and 3 in the vertical direction, respectively, and θ_1, θ_2, and θ_3 denote the rotation of masses 1, 2, and 3 from a fixed reference direction. F is the force exerted on mass 3 and T is the torque exerted on mass 3, as shown in the figure. All coordinates are measured from the equilibrium position defined by $F = 0$ and $T = 0$. The coefficients $k, \delta,$ and ϵ are positive constants characterizing the Wilberforce springs.

The system of equations (1) is linear and can be written in the form

$$
Au = b, \tag{2}
$$

where $u = (x_1\ \ \theta_1\ \ x_2\ \ \theta_2\ \ x_3\ \ \theta_3)^{\mathrm{T}}$, $b = (0\ \ 0\ \ 0\ \ 0\ \ F\ \ T)^{\mathrm{T}}$ and A is a 6×6 matrix.

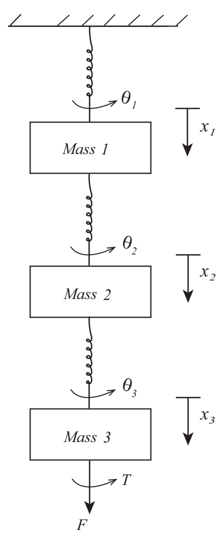

Figure 1: A triple pendulum made of 3 Wilberforce springs that is the subject of Question 4.

PLEASE FIND THE QUESTIONS ON THE NEXT PAGE

Question 4 (continued)

Please answer the following questions

(i) (13 points) Matrix A is

(a) Diagonal (nonzero entries only on the main diagonal)

(b) Tridiagonal

(c) The identity matrix

(d) Full (each entry of A is nonzero)

(e) None of the above

(ii) (12 points) If A_{ij} denotes the element of A in the ith row and jth column, then A_{54} and A_{65} are given by

(a) k and $-\delta$, respectively

(b) 2δ and ϵ, respectively

(c) k and ϵ, respectively

(d) $-\epsilon$ and ϵ, respectively

(e) None of the above