2.087 Differential Equations and Linear Algebra, Spring 2014

Homework #1

Date Issued: Wednesday 3 September, 2014

Date Due: Wednesday 10 September, 2014, 9:30AM (bring hard copy to lecture)

As described in the course policies document, this is one of 5 homeworks you will complete in this course. Each of these count as 6% of your total grade. Full credit can generally only be earned by showing your work. This often includes making clear and well-labeled plots.

1) (5 points) Make a graph of $y = e^t$ for $-1 \le t \le 1$ either by hand or using MATLAB®.

What is the value of the function *y* at *t*=0?

What is the value of the function *y* at *t*=1?

What is the slope dy/dt at *t*=0?

What is the slope dy/dt at *t*=1?

2) (15 points) For each of the differential equations and solutions below, demonstrate that the proposed solution satisfies the differential equation.

a. $2t\dot{y} - 4y + 12 = 0$ Solution: $y(t) = 2t^2 + 3$

b. $t\dot{y} - y(1-t) = t^2$ Solution: $y(t) = te^{-t} + t$

c. $6y - \frac{1}{3}\dot{y}\ddot{y} = 0$ Solution: $y(t) = t^3$

3) (6 points) For each of these differential equations, indicate whether it is linear in *y*.

a. $dy/dt + \sin y = t$

b. $y' = t^2(y - t)$

c. $y' + e^t y = t^{10}$

4) (4 points) What linear differential equation $dy/dt = a(t)y$ is satisfied by $y(t) = e^{\cos(t)}$?

5) (10 points) All solutions of $dy/dt = -y + 2$ approach as steady state where $dy/dt = 0$ and $y = y_\infty$ That value, y_∞ is a particular solution. What null solution $y_n = Ce^{-t}$ combines with the particular solution to satisfy $y(0) = 4$?

6) (10 points) Find the solution of $dy/dt + 2y = 6$ where $y(0) = 1$? What is y_∞ ? Make its graph.

7) (10 points) Draw the function that solves $y' = H(t - T)$ where $y(0) = 2$. Note $H(t)$ is the unit step function.

8) (20 points)

a) Find the function that solves $y' - y = \delta(t - 2)$ where $y(0) = 3$. Note $\delta(t)$ is the delta function.

b) Make a graph of the solution (by hand or with a computer). Comment on any features of the graph that are notable to you.

9) (20 points) A model aircraft is pointed straight down with its engines off. At time t=0 sec, it has just begun descent from a vertical climb maneuver and it has, momentarily, zero airspeed, $V(0\text{s})=0$ m/s. Its mass is 1.2 kg and its weight causes acceleration. To determine the effect of aerodynamic drag as speed builds, consider the drag force is given by 1/2 ρ V^2 S C_D where its drag coefficient C_D = 0.02 and the area S is 0.22 m^2 and density of the air ρ is the typical value for sea level about 1.3 kg/ m^3. At time t=5 sec, it deploys speed brakes so its drag coefficient changes suddenly to C_D = 0.08.

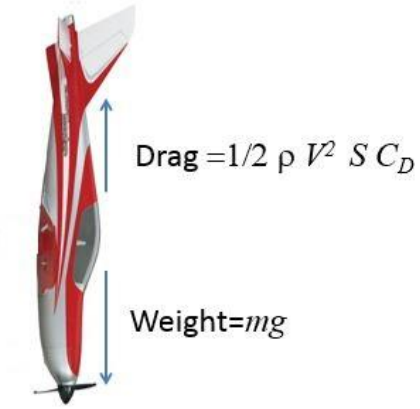
a) Write a differential equation modeling the evolution of airspeed $V(t)$ from t=0 sec to t=5 sec.

b) Find the solution to the equation in (a) satisfying the initial condition $V(0\text{s})=0$ m/s and find the speed at to t=5 sec.

c) Write a differential equation modeling the evolution of velocity from t=5 sec onward and choose a condition to define an "initial" value problem.

d) Find the solution to the equation in (c) or else describe as many features of the solution as you can infer within a reasonable time allocation.

e) Estimate the value of the time t when the aircraft gets to within 5% of the steady state speed after the time that speed brakes were deployed.



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