## 2.087 Differential Equations and Linear Algebra, Spring 2014

# Homework #2

**Date Issued:** Wednesday 10 September, 2014

**Date Due:** Monday 22 September, 2014, 9:30AM (bring hard copy to lecture)

**As described in the course policies document, this is one of 5 homeworks you will complete in this course. Each of these count as 6% of your total grade.** Full credit can generally only be earned by showing your work. This often includes making clear and well-labeled plots.

1) (10 points)   For each of the following differential equations determine: (1) the order of the differential equation, (2) whether the equation is linear or nonlinear, and (3) if the equation is linear, determine whether the equation is homogeneous.

a. $(1+y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$

b. $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} = 1$

c. $\frac{dy}{dt} + ty^2 = 0$

d. (pendulum equation) $\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$, where $g, l$ are parameters

e. (Duffing oscillator equation) $\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} + \alpha x + \beta x^3 = \gamma\cos\omega t$, where $\delta, \alpha, \beta, \gamma$ are parameters

2) (10 points)   Find a formula for $y(t)$ with $y(0)=1$ and draw its graph.  What is $y_\infty$?

a. $y' + 2y = 6$

b. $y' + 2y = -6$

3) (10 points) Find the solutions of these differential equations starting from $y(0)=2$ :

a.  $dy/dt - y = 8e^{3t}$

b.  $y' + y = 8e^{-3t}$     What is $y_\infty$?

4) (10 points) What differential equations $dy/dt = ay + q(t)$ would be solved by $y_1(t)$ and $y_2(t)$ plotted below?

a)



$y_1(t)$

b)



$y_2(t)$

$y_2(t) = e^t - 1$ if $0 \leq t < 1$

$y_2(t) = e^{2-t} - 1$ if $1 \leq t < 2$

$y_2(t) = 0$ if $t \geq 2$ or $t < 0$
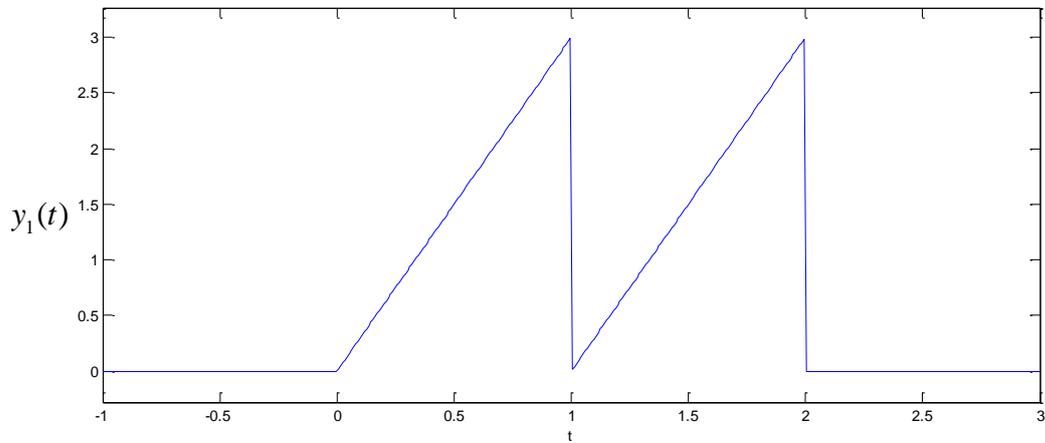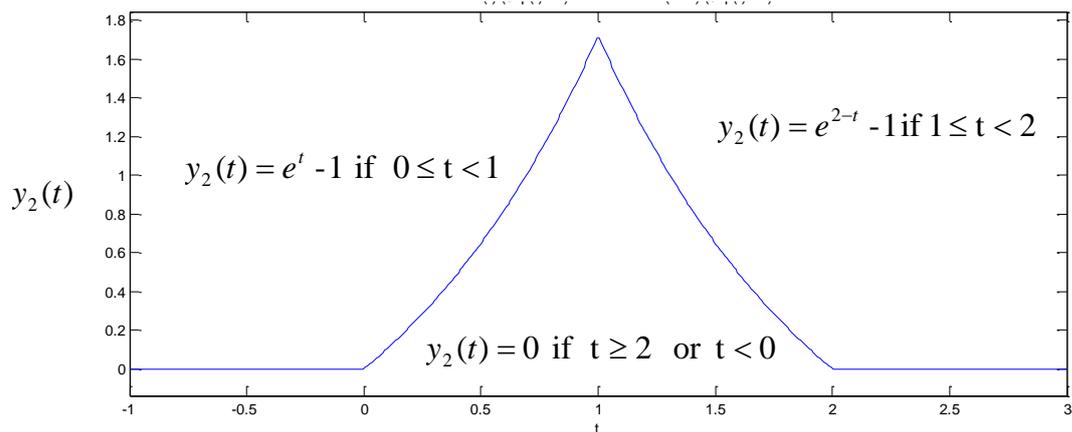
5) (10 points) Find the solution to the differential equation $dy/dt = H(t-1) + H(t-2) + 2H(t-3)$ that satisfies the initial condition $y(0) = 2$. Make a graph the solution $y(t)$ for $-1 < t < 4$.

6) (10 points) Find the solution to $xy^2 y' = x^3 + y^3$. Graph the solution for a domain in which the solution remains finite.

7) (10 points) The Gompertz model for growth in tumors is $y' = -Ay \ln(y)$ where $A > 0$ and $y(t)$. Is the mass of the tumor cells at time $t$. The declining growth rate with increasing $y > 1$ corresponds to the fact that cells in the interior of the tumor die due to insufficient oxygen and nutrients. Use the ODE to discuss the growth and decline of solutions and to find constant solutions. Also, solve the ODE.

8) (15 points) The figure below depicts an RL circuit

$$V(t) = 5\cos(\omega t)$$
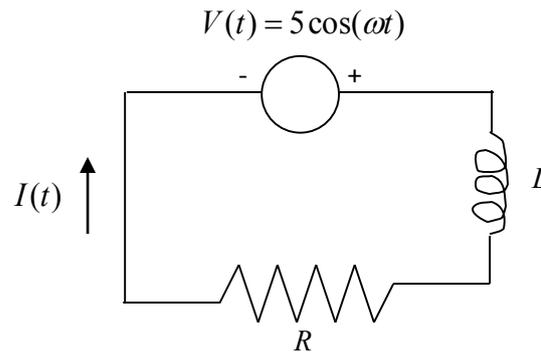


a) Solve $L\,dI/dt + RI(t) = 5\cos(\omega t)$ for the current $I(t)$ in the circuit.

b) One can define a complex impedance $Z = R + i\omega L$ and formulate $I(t) = Ie^{i\omega t}$ in which case, $ZI = V$ and the formula from part (a) becomes:

$$L\,dI/dt + RI(t) = (i\omega L + R)Ie^{i\omega t} = 5e^{i\omega t}$$

What is the magnitude $|Z|$ and the phase angle $\theta$ in $Z = |Z|e^{i\theta}$?

9) (15 points) A vehicle is powered by a two permanent magnet DC electric motors, each driving one of the rear wheels. The torque speed curves of the motors can be characterized by the equation

$$\tau(\Omega) = (K_t I) = \left( K_t \frac{(V - E)}{R} \right) = \left( \frac{K_t V_0}{R} - \frac{K_t^2}{R}\Omega \right)$$

Where $K_t$ is the torque constant (0.011 Nm/V), $V_o$ is the applied voltage, and $R$ is the resistance of the motor's windings (1.1 Ohms). The vehicle has a mass of 0.9 kg centered between the front and rear wheels and is 2 cm above the surface, the wheel base is 15 cm (the distance fore and aft between the wheel axles), the wheel diameter is 4 cm, and the drive train between the motors and the wheels has a gear ratio of 15:1. The wheels and are very light so that their inertia can be neglected along with the inertia of the shaft and drive train. The vehicle begins with zero velocity and the applied voltage $V_o$ is ramped up linearly from zero to 7.4V over the course of 15 seconds. The coefficient of friction between the wheels and the ground can be assumed to be very high so that the wheels do not slip. The rolling resistance of the wheels is very low, so that can be neglected. The velocities are low so that drag can be neglected. Write a differential equation to describe the change in angular velocity of the motor with time $\Omega(t)$ and solve the initial value problem assuming $\Omega(0)=0$ for 0<t<15 sec.

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