
Quiz #1
Open Book, MATLAB[®] Allowed

7 problems in 85 minutes. A sensible pace might be 10 minutes per problem leaving time to check your work. Manage your time accordingly. Show your work as clearly as you can within the time constraints.

1) (20 points) One instance of the logistic equation is $\frac{dy}{dt} = 2y - 3y^2$.

a) Confirm that the equation $y(t) = \frac{2}{de^{-2t} + 3}$ is a solution for any value of d

b) Confirm that the equation $y(t) = \frac{2}{3}$ is a solution

c) Given an initial condition just slightly below $\frac{2}{3}$, like $y(0) = \frac{2}{3} - 0.001$ will the slope

$\frac{dy}{dt}$ of the solution be positive or negative?

Given an initial condition just slightly above $\frac{2}{3}$ like $y(0) = \frac{2}{3} + 0.001$ will the slope $\frac{dy}{dt}$ of the solution be positive or negative?

What is the implication for stability of the state $y(t) = \frac{2}{3}$?

2) (10 points)

a) Find a particular solution to the equation $y' = -2y + 3e^{-x}$.

b) Find a solution to the equation $y' = -2y$.

c) Find a solution to the equation $y' = -2y + 3e^{-x}$ that satisfies the initial condition $y(0) = 2$.

3) (15 points) A solution to a differential equation is known to be

$$y(t) = e^{\sin(\omega t)} + \cos(\omega t) .$$

It is further stipulated that the form of the differential equation is

$$\frac{dy}{dt} = a(t)e^{\sin(\omega t)} + b(t)\sin(\omega t) + ky(t) .$$

Give one valid set of $a(t)$, $b(t)$, and k consistent with this information.

4) (10 points) A function is defined as

$$f(x) = \begin{cases} 0 & x < 0 \\ \sin(x) & 0 \leq x < \pi \\ 1 & \pi \leq x \end{cases}$$

Express the function without recourse to inequality constraints using the Heaviside step function.

5. (10 points)

Consider a solution to the equation $5\dot{y} + y = q(t)$ that satisfies the condition $y(0) = 0$. Call it $y_1(t)$. NOTE: You are not being asked to solve for $y_1(t)$.

Consider a solution to the equation $5\dot{y} + y = s(t)$ that satisfies the condition $y(0) = 0$. Call it $y_2(t)$. NOTE: You are not being asked to solve for $y_2(t)$.

Find a solution to the equation $5\dot{y} + y = 2q(t) - s(t)$ that satisfies the condition $y(0) = 0$. Express it in terms of $y_1(t)$ and $y_2(t)$.

6. (20 points)

A 100 liter tank of a solution of sugar and water has sugar concentration $C(t)$ which has the initial value $C(0)=17$ grams/liter. A flow of another solution of sugar and water with a concentration 3 grams/liter is flowing into the top of the tank at a volumetric flow rate $\dot{V} = 1$ liters per second. A flow of the tank's contents (which are well mixed and therefore the same concentration $C(t)$ exists at all locations in the tank) is emerging out of the bottom of the tank at a volumetric flow rate $\dot{V} = 1$ liters per second.

- a) Write a differential equation relating concentration $C(t)$ and its derivative $\frac{dC}{dt}$. Is the equation linear or non-linear?

- b) Solve the differential equation to find $C(t)$ that satisfies the stipulated initial condition $C(0)= 17$ grams/liter.

- c) Sketch the solution $C(t)$ either based on the result from (b) or based on your understanding of how the system should behave over time or both.

- d) Consider the time at which the concentration reaches $C(T)=10$ grams/liter which you will note is the arithmetic average of the initial tank concentration and the concentration of the inflowing solution. QUESTION: Is the time T greater than, less than, or equal to the time ($t=50$ sec) which is the time that 50 liters of new fluid have entered the 100 liter tank?

7. (15 points)

a) Find a solution to the equation $y' = -3y + t^2\delta(t - 4)$ that satisfies the condition $y(1) = 2$.

b) Graph the solution from (a) over the interval $0 \leq t \leq 5$ and comment on whether the complete solution from (a) makes sense in light of the null and particular solutions.

MIT OpenCourseWare
<http://ocw.mit.edu>

2.087 Engineering Math: Differential Equations and Linear Algebra
Fall 2014

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.