2.094 — Finite Element Analysis of Solids and Fluids	Fall '08

Lecture 1 - Large displacement analysis of solids/structures

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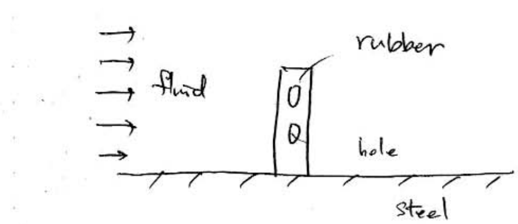
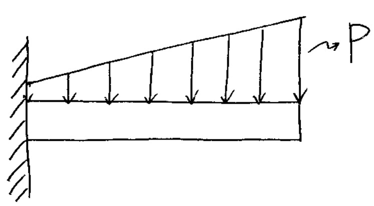
1.1 Project Example

Physical problem

Reading: Ch. 1 in the text

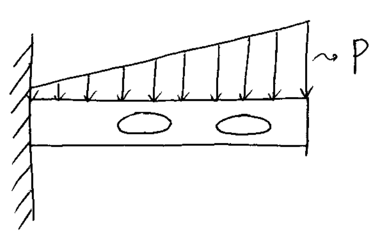


"Simple" mathematical model



- analytical solution
- F.E. solution(s)

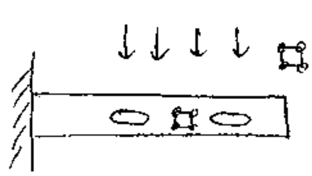
More complex mathematical model



- holes included
- large disp./large strains
- F.E. solution(s) ⇒
- How many finite elements?

We need a good error measure (especially for FSI)

"Even more complex" mathematical model



The "complex mathematical model" includes Fluid Structure Interaction (FSI).

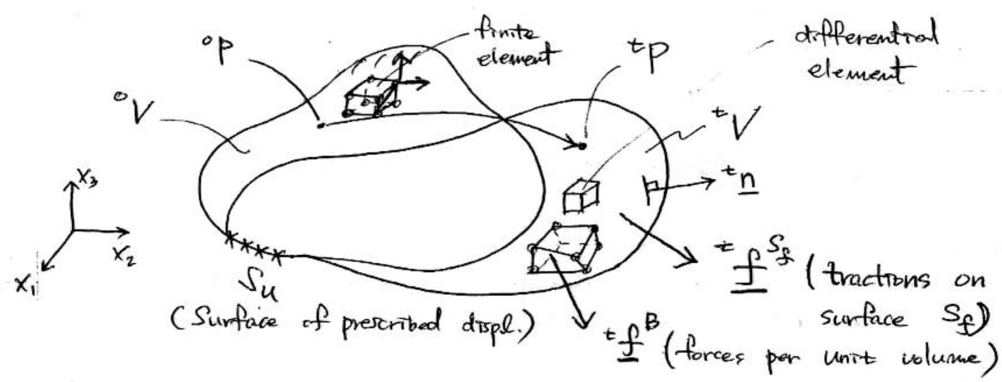
You will use ADINA in your projects (and homework) for structures and fluid flow.

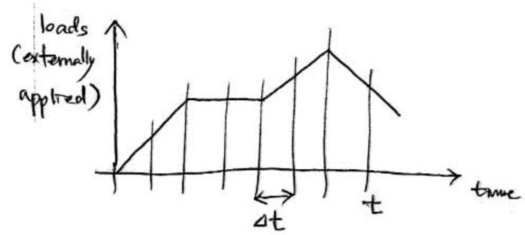
1.2 Large Displacement analysis

Lagrangian formulations:

- Total Lagrangian formulation
- Updated Lagrangian formulation

Reading: Ch. 6





1.2.1 Mathematical model/problem

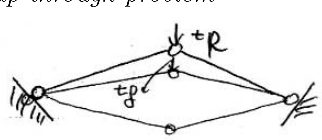
Given the original configuration of the body,
the support conditions,
the applied external loads,
the assumed stress-strain law

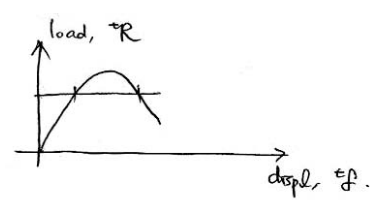
Calculate the deformations, strains, stresses of the body.

Question Is there a unique solution? Yes, for infinitesimal small displacement/strain. Not necessarily for large displacement/strain.

For example:

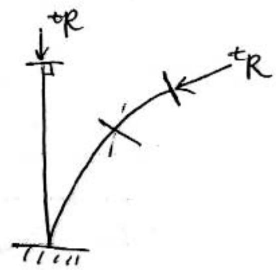
Snap-through problem



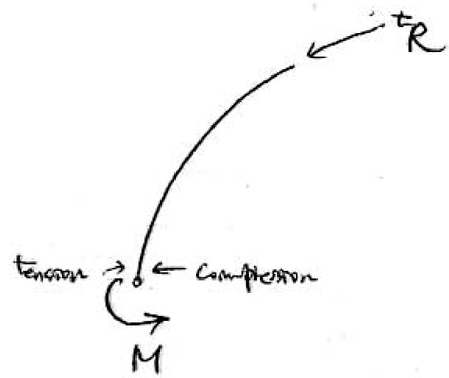


The same load. Two different deformed configurations.

Column problem, statics



Not physical



${}^tR$ is in "direction" of bending moment $\Rightarrow$ Not in equilibrium.

1.2.2 Requirements to be fulfilled by solution at time t

I. Equilibrium of stresses (Cauchy stresses, forces per unit area in ${}^tV$ and on ${}^tS_f$) with the applied body forces ${}^t\boldsymbol{f}^B$ and surface tractions ${}^t\boldsymbol{f}^{S_f}$

II. Compatibility

III. Stress-strain law

1.2.3 Finite Element Method

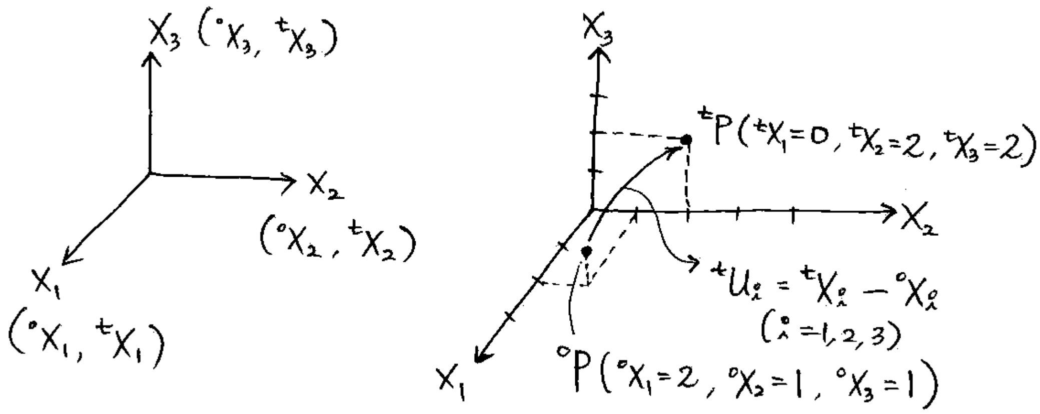
I. Equilibrium condition means now

- equilibrium at the nodes of the mesh
- equilibrium of each finite element

II. Compatibility satisfied exactly

III. Stress-strain law satisfied exactly

1.2.4 Notation



Cauchy stresses (force per unit area at time t):

$$ {}^t\tau_{ij} \quad i,j=1,2,3 \quad {}^t\tau_{ij} = {}^t\tau_{ji} \tag{1.1}$$

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