

2.14/2.140 Problem Set 4

Assigned: Wed. Feb. 26, 2014

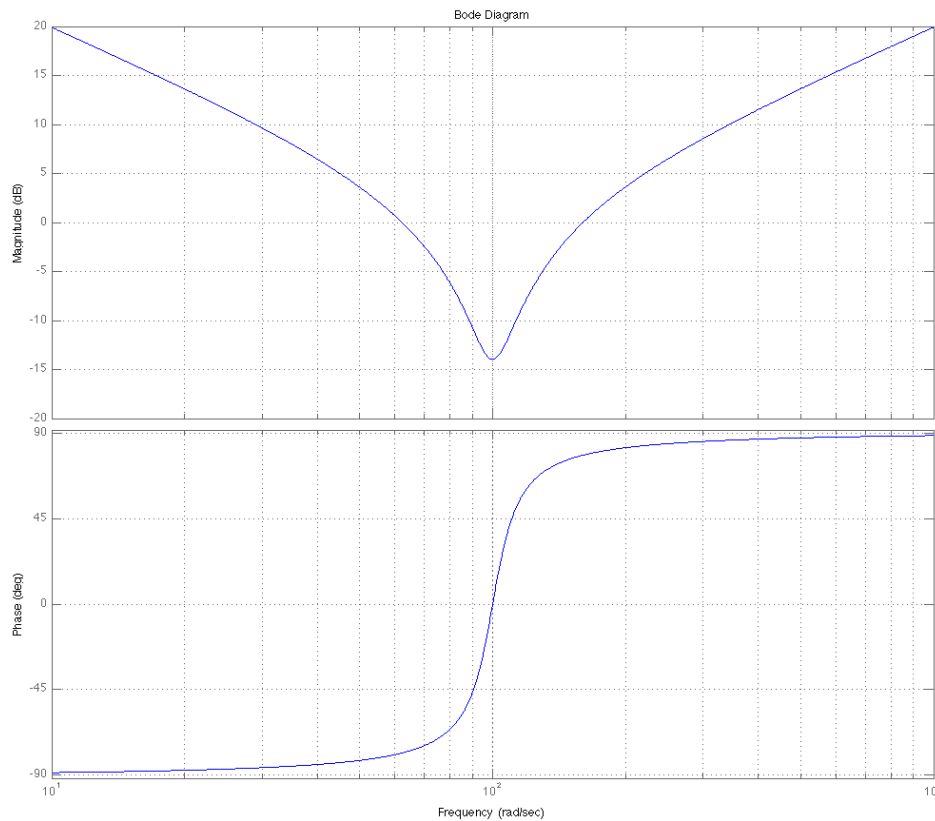
Due: Wed. March 5, 2014, in class

Reading: Lecture notes on loop shaping, frequency response, and Bode plots.

The following problems are assigned to both 2.14 and 2.140 students.

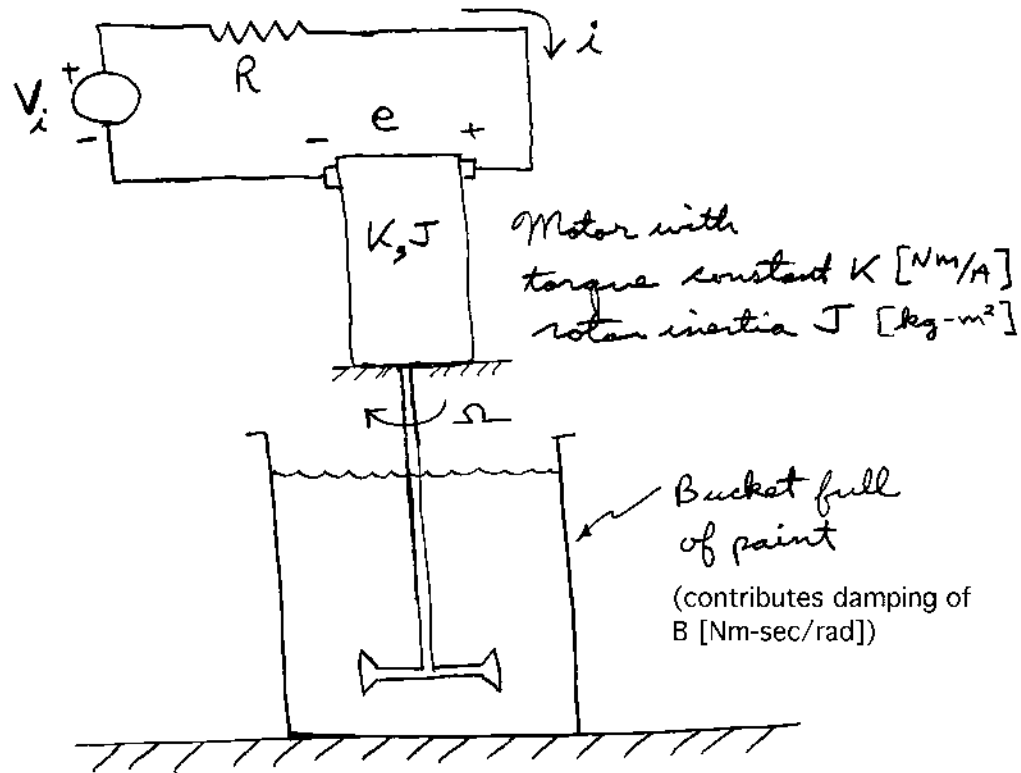
Problem 1 Problem Archive 11.5 Pole Zero Plots. Sketch the indicated Bode plots by hand using the sketching rules presented in the lecture and notes. Do not use Matlab to generate these plots.

Problem 2 The Bode plot given below is for a *notch filter*, which can be used as part of compensation for attenuating resonances. This is called a notch filter because it cuts out a range of frequencies. On the given Bode plot, mark the straight asymptotes, and the phase staircase function. Use this analysis to determine the filter transfer function and draw a pole-zero map for the filter.



Problem 3 Archive Problem 13.1 Rooftop Antenna

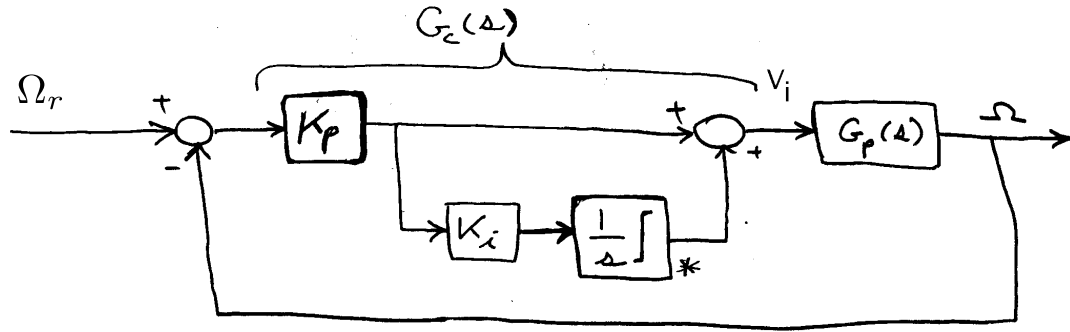
Problem 4 This problem considers the paint stirrer shown below. A brushed DC motor is used to stir a bucket of paint as shown in the figure. The motor has a torque constant K [Nm/A] and the motor plus stirrer inertia is J [kg-m²]. The motor is considered ideal in that it has no inductance and no resistance; the only resistance in the electrical circuit is R as shown. The rotor is rigidly linked to the stirrer, which has no flexibility; the combined assembly has a rotational velocity Ω rad/sec. The effect of the paint on the stirrer is modeled as a rotational damper B [Nms/rad].



- a) Solve for the transfer function $G_p(s) = \Omega(s)/V_i(s)$ in terms of the system parameters. Now set the parameter values as $K = 0.5$ Nm/A, $R = 5$ Ω , $B = 0.15$ Nms/rad, and $J = 0.2$ kgm². For these parameter values, make a hand sketch of the Bode plot for G_p .
- b) Now, let the system input V_i be driven by a PI controller of the form

$$G_c(s) = K_p \left(1 + \frac{K_i}{s} \right).$$

This system has a speed reference input Ω_r and output Ω as shown in the block diagram below



* integrator with saturation

Choose the controller gain values K_p and K_i to set the loop crossover frequency $\omega_c = 10$ rad/sec, with a phase margin $\phi_m = 45^\circ$. Show your calculations. You should be able to accomplish this design using hand-sketched Bode plots, and then using Matlab for confirmation.

- c) Create a Simulink simulation of the loop with the controller implemented as shown in the block diagram above. For now, let the integrator have an unbounded output (leave the integrator limit output box unchecked). Set your simulation to use the variable step solver with a max step size limit of 0.01 sec, and use the ode45 solver. Note that the standard simulation window has a simulation stop time of 10 sec. You may want to adjust this value.

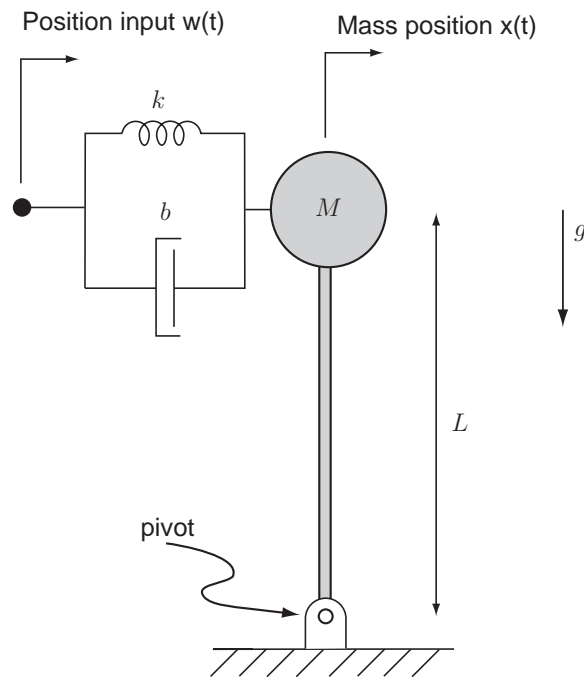
Let the input reference Ω_r take a step from zero to 10 rad/sec at $t = 0$, from initial rest conditions. Run this as a Simulink simulation and record and plot the responses $V_i(t)$ and $\Omega(t)$. What is the maximum value of the V_i during this transient? What is the steady-state value of V_i after the transient has settled?

- d) Now set the integrator limits to ± 1.5 times the steady-state value of V_i from part c), and check the integrator limit box. (This insures that the integrator term is able to supply the required steady-state control effort for a speed of 10 rad/sec, but not much more.) Again let the input reference Ω_r take a step from zero to 10 rad/sec at $t = 0$, from initial rest conditions. Run this as a Simulink simulation and record and plot the responses $V_i(t)$ and $\Omega(t)$. How do these differ from the unlimited case in part c)?
- e) Be sure to keep a copy of your calculations and this model, as we may continue studying it in the next problem set.

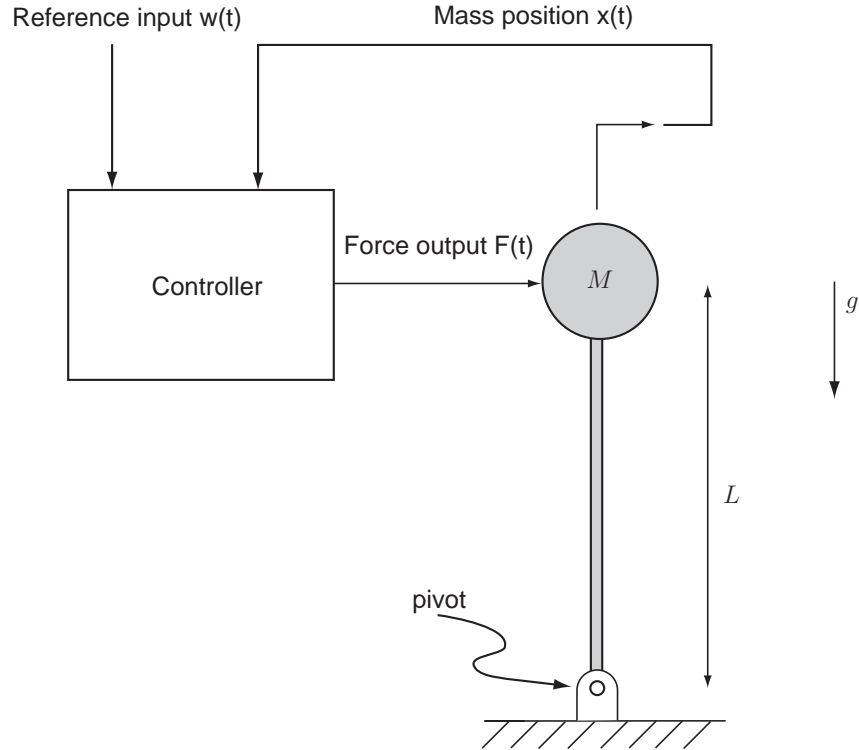
Problem 5 Archive Problem 17.21 Pole Zero Bode Matching

The following problems are assigned to only 2.140 students. Students in 2.14 are welcome to work these, but no extra credit will be given.

Problem G1 This problem was used as the ME quals systems oral exam in 2003. In the inverted pendulum system shown below, note that gravity acts in the downward direction as shown. A rigid massless link constrains the mass M to rotate about the pivot shown. The distance from the pivot point to the center of gravity of the mass is L m. The mass has negligible moment of inertia. The horizontal position of the mass in meters is $x(t)$. The system is driven by a position input $w(t)$ m. The input is connected to the mass via the spring k with units of N/m, and via the damper b shown with units of Ns/m. In the problem, we only consider models which describe motions in which x is very small; do not attempt to write models for large motions.



- For this system, derive a mathematical model which describes the system for **small motions**. The resulting linear model should have an input $w(t)$ and output $x(t)$. Be sure to take into account the effect of gravity. Explain your reasoning. What is the transfer function $X(s)/W(s)$?
- For what range of k and b is the system stable?
- Now consider the controlled pendulum system shown below.



The controller has inputs of the mass position $x(t)$ and reference command $w(t)$. The controller outputs a force $F(t)$ which is applied to the mass as shown. Develop a controller which provides an identical small-motion transfer function $X(s)/W(s)$ to that derived in part a) above. You can feel free to use any linear operation within the controller block. Explain your reasoning for the controller you develop.

- d) Now assume that a time delay of T sec is introduced into the position measurement before it is received by the controller. How does this affect the stability of the controlled system? Why? Show your analytical reasoning. Hint: An approach looking at phase margin is the best way to understand the effect of the time delay.

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2.14 / 2.140 Analysis and Design of Feedback Control Systems
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