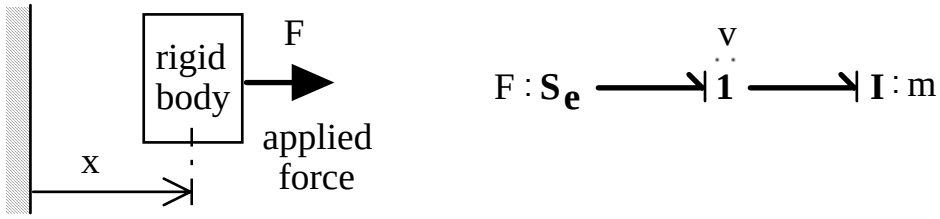
State Variables and Energy

Causality assignment procedures may be used to identify a minimum set of variables necessary to define the energetic state of a system. However, it is important to recognize that everything of interest about a system will not always be determined by its energetic state. We will frequently need to choose state variables in addition to those associated with independent energy storage elements. This is illustrated in the following example.

Example: A Trivial Mechanical System



(a) (b)

Figure 6.3

Consider a rigid body subject to an applied force as depicted in figure 6.3a. A corresponding bond graph with casual assignment is shown in figure 6.3b. There is a single independent energy storage element in this system and its energetic state is determined by a single variable, either the momentum of the rigid body which determines the kinetic energy

$$E_k = \frac{p^2}{2m} \tag{6.37}$$

or the coresponding speed which determines the kinetic co-energy

$$E_k^* = \frac{1}{2}\, mv^2 \tag{6.38}$$

A corresponding state equation is as follows.

$$\frac{d}{dt}\, v = \frac{F}{m} \tag{6.39}$$

However, we will often be concerned with the location of the rigid body, not just with its speed or energy. In that case we will need an additional state variable and a suitable set of state equations is as follows.

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F \tag{6.40}$$

By a similar argument, in other cases we may need other state variables (e.g. the time integral of position, etc.). Thus the state variables identified by the causality assignment procedures are merely a minimum set required to define the system energy.

No Fixed Rules!

To summarize: there are no fixed rules for choosing state variables. Using the substitution methods reviewed above, the following guidelines are offered: generally, for systems with linear energy-storage elements, power variables are appropriate; for systems with nonlinear energy-storage elements, energy variables are recommended. For mechanical systems, Lagrangian variables are recommended. Most important, use variables that make the most physical sense to you.