Ideal Dissipative Elements

Energy cannot be destroyed, but it can be lost or dissipated, irreversibly removed from a system. Loss or dissipation of energy may be characterized by an ideal dissipative element, ideal in the sense that it does not store, supply or transmit energy, but simply removes it from the system. The bond graph symbol is:

Figure 3.12: Bond graph symbol for an ideal dissipative element.

Any phenomenon characterized by an algebraic relation (possibly nonlinear) between effort and flow will exhibit this behavior, provided the relation exists only in the first and third quadrants, as indicated in the following diagram.

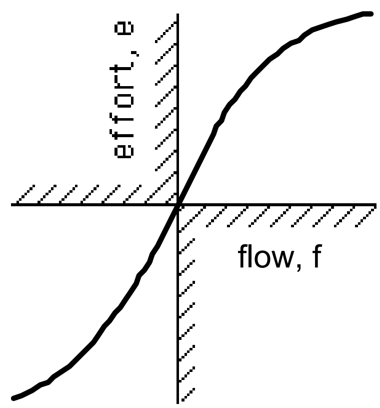

Figure 3.13 Sketch of a possible ideally dissipative characteristic.

The restriction to the first and third quadrants ensures that the product of effort and flow is never negative[1], and that, by our sign convention, ensures that power always flows into the element and never out of it; this is a passive element. We may augment the bond graph with a half-arrow denoting the power sign convention.

Figure 3.14: Bond graph symbol for an ideal dissipator with sign convention.

[1] Here we are assuming the usual convention that positive values are upwards on the vertical axis and to the right on the horizontal axis.

Dissipative elements come in either of two causal forms. If the dissipator accepts flow as input, it has *resistance causality.* An *ideal resistor* is defined as an element for which effort is a single-valued algebraic function of flow.

$$e = R(f) \tag{3.13}$$

Adding the causal stroke to the bond graph:

Figure 3.15: Bond graph for an ideal dissipator with sign convention and resistance causality.

The algebraic function R(·) is the *constitutive equation* for this element. The term refers to the fact that this equation describes and is defined by the physical constitution of the device or phenomenon being modeled. Using the constitutive equation, the power into this element may be written as a function of the input flow alone.

$$P(f) = R(f)\, f \tag{3.14}$$

Thus the power dissipated by this element is a function of its input, which is determined by the rest of the system.

An ideal resistor may have a nonlinear constitutive relation. An ideal linear resistor is defined to have a linear constitutive relation.

$$e = R\, f \tag{3.15}$$

The power flow into an ideal linear resistor is a quadratic function of the input flow.

$$P(f) = R\, f^2 \tag{3.16}$$

The parameter R is the resistance of this ideal linear dissipative element. In bond graph notation, parameter values may be written next to the corresponding symbol as shown in the next figure[2], but this is optional.

Figure 3.16: Bond graph convention for denoting parameter values.

[2] This apparently redundant practice will make more sense when we need to distinguish between similar types of element, e.g. resistors, with different parameter values, e.g. R_1, R_2, etc.

An example of an ideal linear resistor is the familiar (idealized) electrical resistor, characterized by Ohm's law, which states that the voltage drop, e, across the resistor (an effort) is proportional to the current, i, through the resistor (a flow).

$$e = R\,i \tag{3.17}$$

Because the ideal linear electrical resistor is the commonest example of a dissipative element, an ideal (nonlinear) dissipator is sometimes called a *generalized* resistor.

If the dissipator has the dual causal form and accepts effort as input it has *conductance causality.* This is an element which has a constitutive equation defining flow as a single-valued algebraic function of effort.

$$f = G(e) \tag{3.18}$$

Adding the causal stroke to the bond graph:

Figure 3.17: Bond graph for an ideal dissipator with sign convention and conductance causality.

The power flow into this element may be written as a function of the input effort alone.

$$P(e) = e\,G(e) \tag{3.19}$$

An ideal linear conductance has a linear constitutive relation.

$$f = G\,e = e/R \tag{3.20}$$

The power flow into this element is a quadratic function of the input effort.

$$P(e) = G\,e^2 = e^2/R \tag{3.21}$$

The parameter G = 1/R is the conductance of this ideal linear dissipative element. Again, it may optionally be written next to the symbol for the element as shown in figure 3.18.

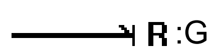

Figure 3.18: Dissipator in conductance causality with associated parameter.

Causal Constraints

A word of caution is appropriate at this point. If we are dealing with a linear element, we are assured that its constitutive equation can be inverted, (with the possible exception of the degenerate cases R = 0 or G = 0) and so the element can assume either causal form. A linear dissipator can be modelled equally well as a resistance or as a conductance; the choice of which may be determined by considering other elements in the system.

For a nonlinear element the matter may not be so simple. Frequently, a nonlinear constitutive equation will have a well-defined inverse (as in the sketch in figure 3.13), but this will not always be the case. Consider, for example, Coulomb's friction law, commonly used to represent dry friction.

$$F_{friction} = B \, \mathrm{sgn}(v) \tag{3.22}$$

where B is a constant and sgn(·) is the signum function.

$$\mathrm{sgn}(v) \triangleq \begin{cases} 1 \text{ if } v > 0 \\ 0 \text{ if } v = 0 \\ -1 \text{ if } v < 0 \end{cases} \tag{3.23}$$

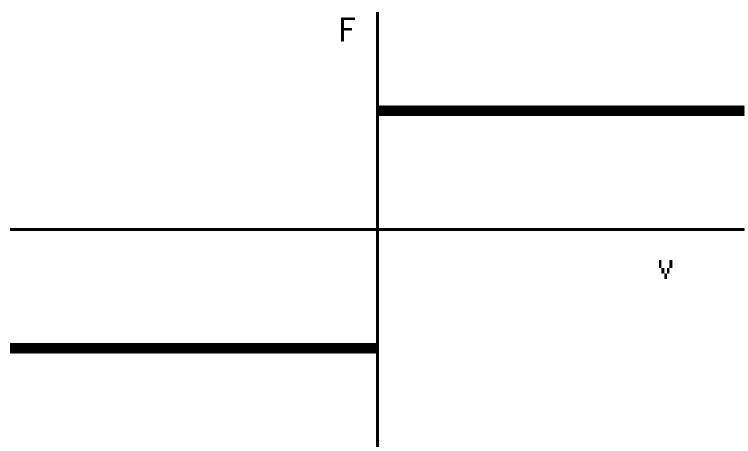

Figure 3.19: Dissipative characteristic of a Coulomb friction element.

Equating force with effort and speed with flow as in Table 3.1, we can see that this object is an ideal dissipator which has resistance causality. An appropriate bond graph would be figure 3.15 and its constitutive equation is depicted in the figure 3.19.

However, this constitutive equation cannot be inverted. Whereas there is an unique force corresponding to every speed, there is only one value of force, F = 0, which determines an unique speed; two values of force, F = +B and F = -B correspond to infinite sets of speeds ($v > 0$ and $v < 0$ respectively); and all other values of force do not correspond to any defined speed at all. This model cannot be represented in conductance causality.

If the nonlinear constitutive equation of a dissipator is not invertible, the element is *causally constrained.* It <u>must</u> be represented in whichever causal form will result in a well-defined function which operates on the input to produce an unique output.