Momentum-based fluid-mechanical power transduction.

A domestic vacuum cleaner is a good example of a consumer product that fundamentally involves power transduction between the electrical, mechanical and fluid domains. How should the transduction mechanism of the pump be modeled? The best answer is based on understanding the basic fluid-mechanical transduction phenomenon in a turbine or centrifugal pump. This is due to a change of fluid momentum which requires a force or torque. Thus fluid flow is related to rotational effort; this transduction phenomenon may be modeled by a gyrator.

Suppose you didn't see this; how could you deduce it? One way is to consider what is required to reproduce observable behavior of the vacuum cleaner. A key clue is the observation that when the air hole of a vacuum cleaner is blocked the vacuum cleaner *speeds up.* Let's try to model this phenomenon.

Motor: Assume no significant electrical dynamics; model the electric motor as a torque source; include motor inertia, bearing friction.

$$I:J_m$$

$$Se \underset{\tau_m}{} \text{———} 1 \text{———} \boxed{to\ pump}$$

$$R:B_m$$
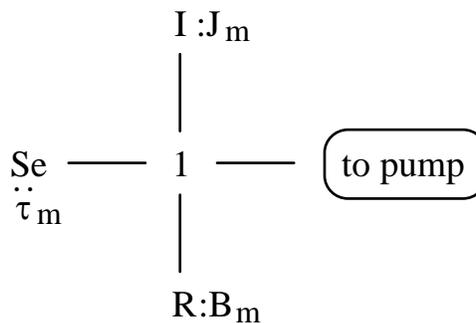
Fluid flow: Include orifice resistance, neglect hose capacitance and fluid inertia.

$$\boxed{from\ pump} \text{———} R:R_{orifice}$$
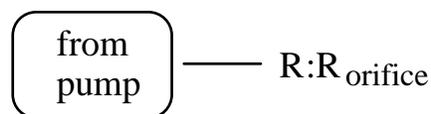
First, try modeling the pump as a transformer. The blocked flow boundary condition may be represented as a flow source with zero flow as follows.
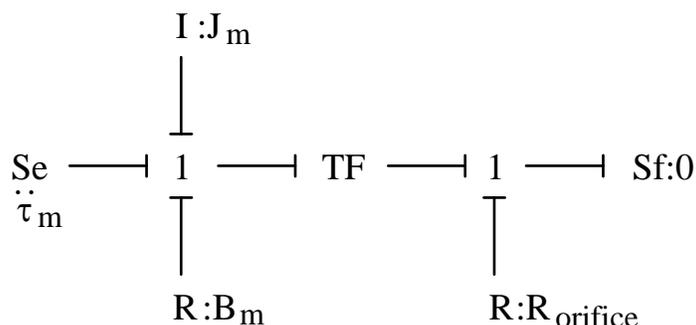
$$I:J_m$$

$$Se \underset{\tau_m}{} \text{———} 1 \text{———} TF \text{———} 1 \text{———} Sf:0$$

$$R:B_m \qquad\qquad R:R_{orifice}$$

Propagating causality, it can be seen that this model would predict that blocking the air hole would stall the motor.

Now try modeling the pump as a gyrator:

$$I:J_m$$

$$Se \overset{..}{\tau}_m \longrightarrow 1 \longmapsto GY \longmapsto 1 \longmapsto Sf:0$$

$$R:B_m \qquad\qquad R:R_{orifice}$$
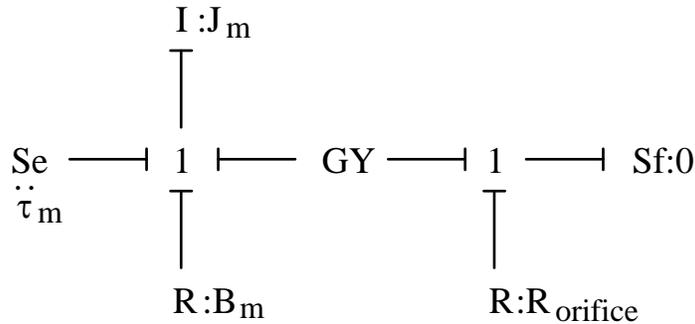
In this case, propagating causality, it can be seen that this model would predict that blocking the air hole will not stall the motor. If we remove all bonds with zero power flow (shown in the following as dashed lines) we can see that the effect of blocking the flow is to eliminate the influence of the fluid resistance on the motor; i.e. the load on the motor decreases.

$$I:J_m$$

$$Se \overset{..}{\tau}_m \longrightarrow 1 \ \vdash\text{---}\ GY \ \text{----}\dashv\ 1 \ \text{----}\dashv\ Sf:0$$

$$R:B_m \qquad\qquad R:R_{orifice}$$
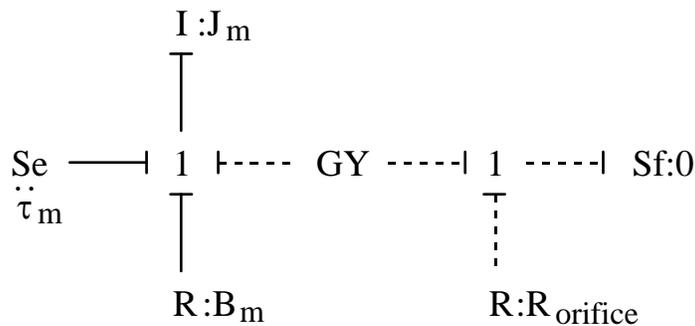
Influence of leakage flow

One might reasonably argue that this marked difference between the predictions of the two models of the pump is an artifact of neglecting the possibility of leakage flow through the pump. If the motor were immobilized, it would still be possible to move air through the pump, yet the transformer model above would predict otherwise. To rectify that error we might include a leakage flow in the fluid flow model as follows:

$$\boxed{\begin{array}{c} \text{from} \\ \text{pump} \end{array}} \longrightarrow 0 \longrightarrow R:R_{orifice}$$

$$R:R_{leakage}$$
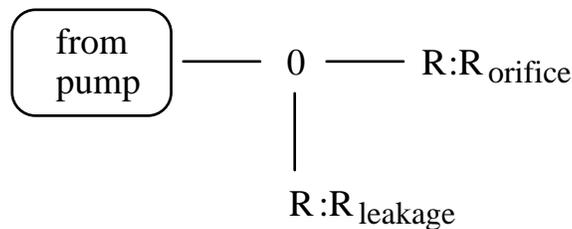
Now if we model the pump as a transformer and impose a blocked-flow boundary condition we find that the motor is not immobilized. Removing all bonds with zero flow (shown as dashed lines) we find that blocked flow eliminates the influence on the motor of orifice resistance but not leakage resistance.

$$\begin{array}{c}
I:J_m \\[4pt]
\vert \\
Se \longmapsto 1 \longmapsto TF \longmapsto 0 \;\text{-----}\; 1 \;\text{-----}\; Sf{:}0 \\
\ddot{\tau}_m \qquad \vert \qquad\qquad \vert \qquad\qquad \vdots \\
R:B_m \qquad R_{leakage}{:}R \qquad R:R_{orifice}
\end{array}$$
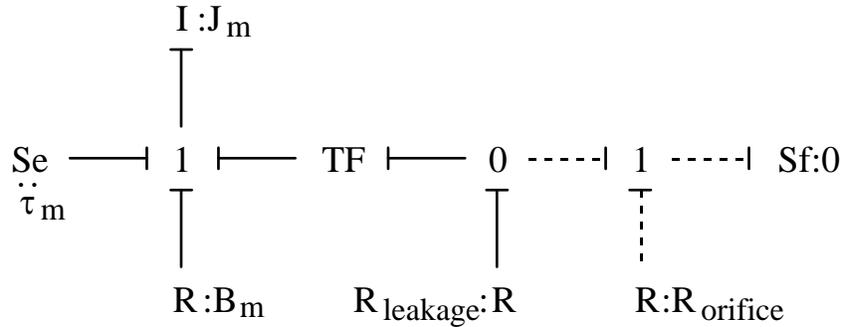
Modeling the pump as a gyrator yields a superficially similar result.

$$\begin{array}{c}
I:J_m \\[4pt]
\vert \\
Se \longmapsto 1 \longmapsto GY \longmapsto 0 \;\text{-----}\; 1 \;\text{-----}\; Sf{:}0 \\
\ddot{\tau}_m \qquad \vert \qquad\qquad \vert \qquad\qquad \vdots \\
R:B_m \qquad\qquad R \qquad\qquad R
\end{array}$$
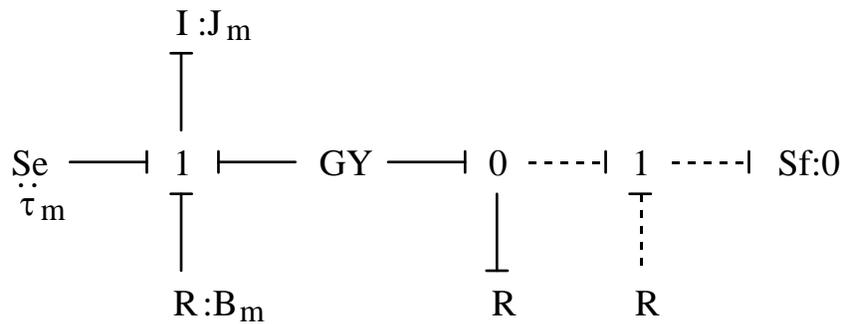
However, the fundamental difference between these two models may be seen by considering the equivalent rotational systems. If the pump is modeled as a transformer and a blocked-flow boundary condition is imposed, the equivalent rotational system is as follows.

$$\begin{array}{c}
I:J_m \\[4pt]
\vert \\
Se \longmapsto 1 \longmapsto 0 \;\text{-----}\; 1 \;\text{-----}\; Sf{:}0 \\
\ddot{\tau}_m \qquad \vert \qquad\quad \vert \qquad\quad \vdots \\
R:B_m \qquad \underset{\begin{subarray}{c} B_{eq.,} \\ leakage \end{subarray}}{R} \qquad \underset{\begin{subarray}{c} B_{eq.,} \\ orifice \end{subarray}}{R}
\end{array}$$
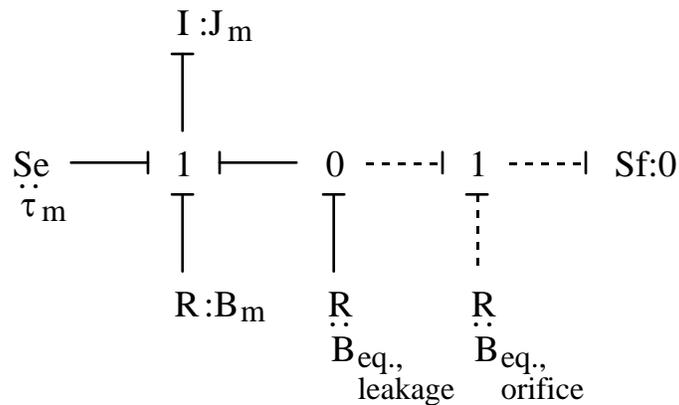
This may be easier to understand and visualize when represented using more conventional graphic symbols for rotational systems. First, without the flow blocked:

J_m

τ_m

B_m $B_{eq.,}$ $B_{eq.,}$
            leakage   orifice
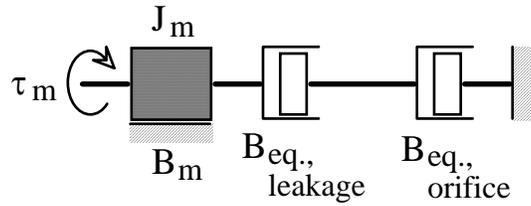
Then with the flow blocked as follows:

blocked flow is equivalent to
immobilizing this component
$\downarrow$

J_m

τ_m

B_m $B_{eq.,}$ $B_{eq.,}$
            leakage   orifice
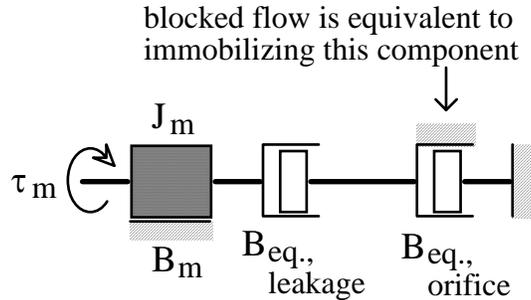
From this we may see that blocking the flow is equivalent to increasing the value of $B_{eq., orifice}$ without limit, which increases the load on the motor.

If the pump is modeled as a gyrator and a blocked-flow boundary condition is imposed, the equivalent rotational system is as follows.

$I:J_m$

Se $\quad$ 1 $\quad\quad$ 1 $\dashmapsto$ 0 $\dashmapsto$ Se:0
τ_m

$R:B_m$ $\quad$ R $\quad\quad$ R
$\quad\quad\quad\quad B^*_{eq.,}$ $\quad B^*_{eq.,}$
$\quad\quad\quad\quad$ leakage $\quad$ orifice
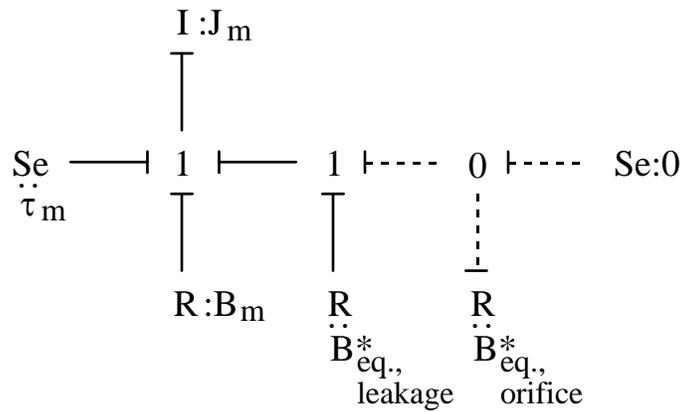
To denote the fact that the equivalent rotational dissipators have different values form the previous case, they have been denoted by $B^*_{eq., leakage}$ and $B^*_{eq., orifice}$ respectively. Again, it may be easier to understand and visualize this using the more conventional graphic symbols for rotational systems. First, without the flow blocked:

J_m $B^*_{eq., \text{orifice}}$

τ_m

$B_m + B^*_{eq., \text{leakage}}$
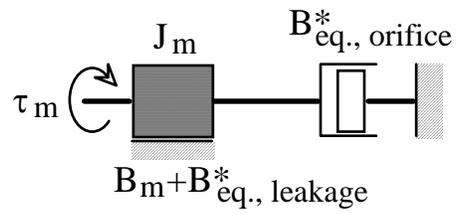
Then with the flow blocked:

blocked flow is equivalent to
disconnecting this component

J_m $B^*_{eq., \text{orifice}}$

τ_m

$B_m + B^*_{eq., \text{leakage}}$
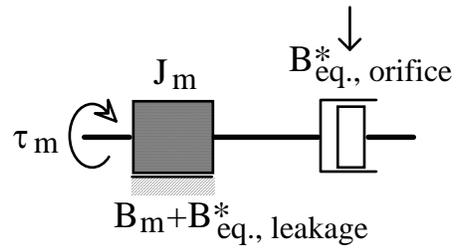
Note that in contrast to the previous case, blocking the flow is equivalent to *decreasing* the value of $B_{eq., \text{orifice}}$ without limit, which decreases the load on the motor.