14 TOWING OF VEHICLES

Vehicles which are towed have some similarities to the vehicles that have been discussed so far. For example, towed vehicles are often streamlined, and usually need good directional stability. Some towed vehicles might have active lifting surfaces or thrusters for attitude control. On the other hand, if they are to be supported by a cable, towed vehicles may be quite heavy in water, and do not have to be self-propelled. The cable itself is an important factor in the behavior of the complete towed system, and in this section, we concentrate on cable mechanics more than vehicle characteristics, which can generally be handled with the same tools as other vehicles, i.e., slender-body theory, wing theory, linearization, etc.. Some basic guidelines for vehicle design are given at the end of this section.

Modern cables can easily exceed $5000m$ in length, even a heavy steel cable with $2cm$ diameter. The cables are generally circular in cross section, and may carry power conductors and

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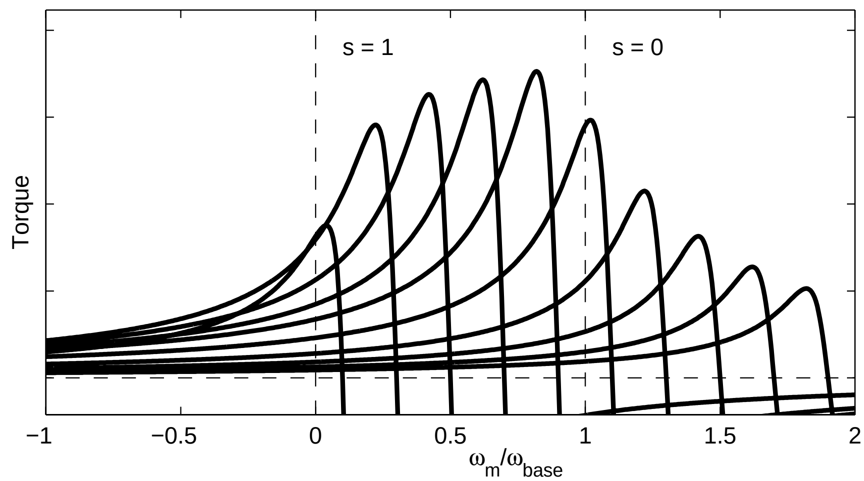


Figure 8: Effects of constant Volts/Hz speed control; voltage is constant above electrical frequency ω_{base}.

multiple communication channels (fiber optic). The extreme L/D ratio for these cables obviates any bending stiffness effects.

Cable systems come in a variety of configurations, and one main division may be made simply of the density of the cable. Light-tether systems are characterized by neutrally-buoyant (or nearly so) cables, with either a minimal vehicle at the end, as in a towed array, or a vehicle capable of maneuvering itself, such as a remotely-operated vehicle. The towed array is a relatively high-velocity system that nominally streams out horizontally behind the vessel. An ROV, on the other hand, operates at low speed, and must have large propulsors to control the tether if there are currents. Heavy systems, in contrast, employ a heavy cable and possibly a heavy weight; the rationale is that gravity will tend to keep the cable vertical and make the deployment robust against currents and towing speed. The heavy systems will generally transmit surface motions and tensions to the towed vehicle much more easily than light-tether systems.

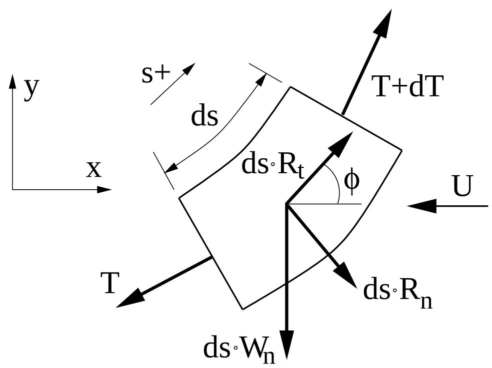
We will not discuss light systems specifically here, but rather look at heavy systems. Most of the analysis can be adapted to either case, however.

14.1 Statics

14.1.1 Force Balance

For the purposes of deriving the static configuration of a cable in a flow, we assume for the moment that that it is inextensible. Tension and hydrostatic pressure will elongate a cable, but the effect is usually a small percentage of the total length.

We employ the curvilinear axial coordinate s, which we take to be zero at the bottom end of the cable; upwards along the cable is the positive direction. The free-body diagram shown



has the following components:

- W_n: net in-water weight of the cable per unit length.
- $R_n(s)$: external normal force, per unit length.
- $R_t(s)$: external tangential force, per unit length.
- $T(s)$: local tension.
- $\phi(s)$: local inclination angle.

Force balance in the tangential and normal coordinates gives two coupled equations for T and ϕ:

$$\frac{dT}{ds} = W_n \sin\phi - R_t \quad (175)$$

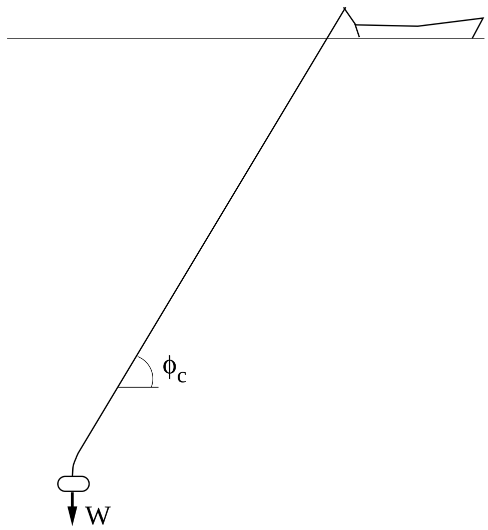
$$T\frac{d\phi}{ds} = W_n \cos\phi + R_n. \quad (176)$$

The external forces are primarily fluid drag; the tangential drag is controlled by a frictional drag coefficient C_t, and the normal drag scales with a crossflow drag coefficient C_n. In both cases, the fluid velocity vector, U horizontal toward the left, is to be projected onto the relevant axes, leading to

$$R_t = -\frac{1}{2}\rho C_t d U^2 \cos^2\phi \quad (177)$$

$$R_n = -\frac{1}{2}\rho C_n d U^2 \sin^2\phi. \quad (178)$$

Note that we simplified the drag laws from a usual form $v|v|$ to v^2, since as drawn, $0 \leq \phi \leq \pi/2$.



The equations for T and ϕ can be integrated along the cable coordinate s to find the cable's static configuration. Two boundary conditions are needed, and the common case is that a force balance on the vehicle, dominated by drag, weight, and the cable tension, provides both $T(0)$ and $\phi(0)$. For example, a very heavy but low-drag vehicle will impose $\phi(0) \simeq \pi/2$, with $T(0)$ equal to the in-water weight of the vehicle.
With regard to Cartesian coordinates x, y, the cable configuration follows

$$\frac{dx}{ds} = \cos\phi \quad (179)$$

$$\frac{dy}{ds} = \sin\phi. \quad (180)$$

The simultaneous integration of all four equations (T, ϕ, x, y) defines the cable configuration, and current dependency may be included, say U is a function of y.

14.1.2 Critical Angle

For very deep systems, the total weight of cable will generally exceed that the vehicle. This gives rise to a configuration in which the cable is straight for a majority of its length, but turns as necessary at the vehicle end, to meet the bottom boundary condition. In the straight part of the cable, normal weight and drag components are equalized. The uniform angle is called the critical angle ϕ_c, and can be approximated easily. Let the relative importance of weight be given as

$$\delta = \frac{W_n}{\rho C_n d U^2},$$

so that the condition $d\phi/ds = 0$ requires from the force balance

$$\delta \cos\phi_c - \frac{1}{2}\sin^2\phi_c = 0.$$

We are considering the case of $0 < \phi_c < \pi/2$. Substituting $\sin^2\phi_c = 1 - \cos^2\phi_c$, we solve a quadratic equation and keep only the positive solution:

$$\cos\phi_c = \sqrt{\delta^2 + 1} - 1. \tag{181}$$

In the case of a very heavy cable, δ is large, and the linear approximation of the square root $\sqrt{1+\epsilon} \approx 1 + \epsilon/2$ gives

$$\begin{aligned} \cos\phi_c &\simeq \frac{1}{2\delta} \longrightarrow \\ \phi_c &\simeq \frac{\pi}{2} - \frac{1}{2\delta}. \end{aligned} \tag{182}$$

For a very light cable, δ is small; the same approximation gives

$$\cos\phi_c \simeq 1 - \delta \longrightarrow \phi_c \simeq \sqrt{2\delta}.$$

The table below gives some results of the exact solution, and the approximations.

δ	exact	$\delta >> 1$	$\delta << 1$
0.1	0.44	-	0.45
0.2	0.61	-	0.63
0.5	0.91	0.57	1.00
1.0	1.14	1.07	1.41
2.0	1.33	1.32	-
5.0	1.47	1.47	-

14.2 Linearized Dynamics

14.2.1 Derivation

The most direct procedure for deriving useful linear dynamic equations for a planar cable problem is to consider the total tension and angle as made up of static parts summed with dynamic parts:

$$T(s,t) = \bar{T}(s) + \tilde{T}(s,t)$$
$$\phi(s,t) = \bar{\phi}(s) + \tilde{\phi}(s,t).$$

We also write the axial deflection with respect to the static configuration as $p(s,t)$, and the lateral deflection $q(s,t)$. It follows that $\tilde{\phi} = \partial q/\partial s$. Now augment the two static configuration equations with inertial components:

$$\begin{aligned} m\frac{\partial^2 p}{\partial t^2} &= \frac{\partial \bar{T}}{\partial s} + \frac{\partial \tilde{T}}{\partial s} - W_n \sin(\bar{\phi} + \tilde{\phi}) - \frac{1}{2}\rho C_t d \left(U\cos\phi + \frac{\partial p}{\partial t} \right)^2 \\ (m+m_a)\frac{\partial^2 q}{\partial t^2} &= (\bar{T} + \tilde{T}) \left(\frac{\partial \bar{\phi}}{\partial s} + \frac{\partial \tilde{\phi}}{\partial s} \right) - W_n \cos(\bar{\phi} + \tilde{\phi}) + \\ & \frac{1}{2}\rho C_n d \left(U \sin\phi - \frac{\partial q}{\partial t} \right)^2 . \end{aligned}$$

Here the material mass of the cable per unit length is m, and its transverse added mass is m_a. Note that avoiding the drag law form $v|v|$ again, we have implicitly assumed that $U\cos\phi > |\partial p/\partial t|$ and $U\sin\phi > |\partial q/\partial t|$. If it is not the case, say $U = 0$, then equivalent linearization can be used for the quadratic drag.
Now we perform the trigonometry substitutions in the weight terms, let $\phi \simeq \bar{\phi}$ for the calculation of drag, and substitute the constitutive (Hooke's) law

$$\frac{\partial \tilde{T}}{\partial s} = EA\frac{\partial^2 p}{\partial s^2}.$$

The static solution cancels out of both governing equations, and keeping only linear terms we obtain

$$\begin{aligned} m\frac{\partial^2 p}{\partial t^2} &= EA\frac{\partial^2 p}{\partial s^2} - W_n \cos\bar{\phi}\frac{\partial q}{\partial s} - \rho C_t d U \cos\bar{\phi}\frac{\partial p}{\partial t} \\ (m+m_a)\frac{\partial^2 q}{\partial t^2} &= \bar{T}\frac{\partial^2 q}{\partial s^2} + EA\frac{\partial p}{\partial s}\frac{\partial \bar{\phi}}{\partial s} + W_n \sin\bar{\phi}\frac{\partial q}{\partial s} - \rho C_n d U \sin\bar{\phi}\frac{\partial q}{\partial t}. \end{aligned}$$

The axial dynamics (p) couples with the lateral equation through the weight term $-W_n \cos\bar{\phi}\tilde{\phi}$. The lateral dynamics ($q$) couples with the axial through the term $\tilde{T}\partial\bar{\phi}/\partial s$. An additional weight term $W_n \sin\bar{\phi}\partial q/\partial s$ also appears. The uncoupled dynamics are both in the form of damped wave equations

$$\begin{aligned} m\frac{\partial^2 p}{\partial t^2} + b_t\frac{\partial p}{\partial t} &= EA\frac{\partial^2 p}{\partial s^2} \\ (m+m_a)\frac{\partial^2 q}{\partial t^2} + b_n\frac{\partial q}{\partial t} &= \bar{T}\frac{\partial^2 q}{\partial s^2} + W_n \sin\bar{\phi}\frac{\partial q}{\partial s}, \end{aligned}$$

where we made the substitution $b_t = \rho C_t d U \cos\bar{\phi}$ and $b_n = \rho C_n d U \sin\bar{\phi}$. To a linear approximation, the out-of-plane vibrations of a cable are also governed by the second equation above.

Because of light damping in the tangential direction, heavy cables easily transmit motions and tensions along their length, and can develop longitudinal resonant conditions (next section). In contrast, the lateral cable motions are heavily damped, such that disturbances only travel a few tens or hundreds of meters before they dissipate. The nature of the lateral response, in and out of the towing plane, is a very slow, damped nonlinear filter. High-frequency vessel motions in the horizontal plane are completely missed by the vehicle, while low-frequency motions occur sluggishly, and only after a significant delay time.

	axial	lateral
wave speed	$\sqrt{\frac{EA}{m}}$ FAST	$\sqrt{\frac{\bar{T}(s)}{m+m_a}}$ SLOW
natural frequency	$\frac{n\pi}{L}\sqrt{\frac{EA}{m}}$	$O\left(\frac{n\pi}{L}\sqrt{\frac{\bar{T}(L/2)}{m+m_a}}\right)$
mode shape	sine/cosine	Bessel function
damping	$C_t \simeq O(0.01)$	$C_n \simeq O(1)$
disturbances travel down cable?	YES	NO

14.2.2 Damped Axial Motion

Mode Shape. The axial direction is of particular interest, since it is lightly damped and forced by the heaving of vessels in seas. Consider a long cable governed by the damped wave equation

$$m\ddot{p} + b_t\dot{p} = EAp'' \tag{183}$$

We use over-dots to indicate time derivatives, and primes to indicate spatial derivatives. At the surface, we impose the motion

$$p(L,t) = P\cos\omega t, \tag{184}$$

while the towed vehicle, at the lower end, is an undamped mass responding to the local tension variations:

$$EA\frac{\partial p(0,t)}{\partial s} = M\ddot{p}(0,t). \tag{185}$$

These top and bottom behaviors comprise the boundary conditions for the wave equation. We let $p(s,t) = \tilde{p}(s)\cos\omega t$, so that

$$\tilde{p}'' + \left(\frac{m\omega^2 - i\omega b_t}{EA}\right)\tilde{p} = 0. \tag{186}$$

This admits the solution $\tilde{p}(s) = c_1 \cos ks + c_2 \sin ks$, where

$$k = \sqrt{\frac{m\omega^2 - i\omega b_t}{EA}}. \tag{187}$$

Note that k is complex when $b_t \neq 0$. The top and bottom boundary conditions give, respectively,

$$\begin{aligned} P &= c_1 \cos kL + c_2 \sin kL \\ 0 &= c_1 + \delta c_2, \end{aligned}$$

where $\delta = EAk/\omega^2 M$. These can be combined to give the solution

$$\tilde{p} = P \frac{\delta \cos ks - \sin ks}{\delta \cos kL - \sin kL}. \tag{188}$$

In the case that $M \to 0$, the scalar $\delta \to \infty$, simplifying the result to $\tilde{p} = P \cos ks / \cos kL$.

Dynamic Tension. It is possible to compute the dynamic tension via $\tilde{T} = EA\tilde{p}'$. We obtain

$$\tilde{T} = -EAPk \frac{\delta \sin ks + \cos ks}{\delta \cos kL - \sin kL}. \tag{189}$$

There are two dangerous situations:

- The maximum tension is $\bar{T} + |\tilde{T}|$ and must be less than the working load of the cable. This is normally problematic at the top of the cable, where the static tension is highest.
- If $|\tilde{T}| > \bar{T}$, the cable will unload completely and then reload with extremely high impulsive forces. This is known as snap loading; it occurs primarily at the vehicle, where $\bar{T}$ is low.

Natural Frequency. The natural frequency can be found by letting $b_t = 0$, and investigating the singularity of $\tilde{p}$, for which $\delta \cos kL = \sin kL$. In general, $kL << 1$, but we find that a first-order approximation yields $\omega = \sqrt{EA/LM}$, which is only a correct answer if $M >> mL$, i.e., the system is dominated by the vehicle mass. Some higher order terms need to be kept. We start with better approximations for sin() and cos():

$$\delta \left(1 - \frac{(kL)^2}{2}\right) = kL \left(1 - \frac{(kL)^2}{6}\right).$$

Employing the definition for δ, and recalling that $\omega^2 = k^2 EA/m$, we arrive at

$$\frac{mL}{M} \left(1 - \frac{(kL)^2}{2}\right) = (kL)^2 \left(1 - \frac{(kL)^2}{6}\right).$$

If we match up to second order in kL, then

$$\omega = \sqrt{\frac{EA/L}{M + mL/2}}.$$

This has the familiar form of the square root of a stiffness divided by a mass: the stiffness of the cable is EA/L, and the mass that is oscillating is $M + mL/2$. In very deep water, the effects of $mL/2$ dominate; if $\rho_c = m/A$ is the density of the cable, we have the approximation

$$\omega \simeq \frac{1}{L}\sqrt{\frac{2E}{\rho_c}}.$$

A few examples are given below for a steel cable with $E = 200 \times 10^9 Pa$, and $\rho_c = 7000 kg/m^3$. The natural frequencies near wave excitation at the surface vessel must be taken into account in any design or deployment. Even if a cable can withstand the effects of resonance, it may be undesirable to expose the vehicle to these motions. Some solutions in use today are: stable vessels (e.g., SWATH), heave compensation through an active crane, a clump weight below which a light cable is employed, and an S-shaped length of cable at the bottom formed with flotation balls.

$L = 500m$	$\omega_n = 15.0 rad/s$
$1000m$	$7.6 rad/s$
$2000m$	$3.7 rad/s$
$5000m$	$1.5 rad/s$

14.3 Cable Strumming

Cable strumming causes a host of problems, including obvious fatigue when the amplitudes and frequencies are high. The most noteworthy issue with towing is that the vibrations may cause the normal drag coefficient C_n to increase dramatically – from about 1.2 for a non-oscillating cable to as high as 3.5. This drag penalty decreases the critical angle of towing, so that larger lengths of cable are needed to reach a given depth, and the towed system lags further and further behind the surface vessel. The static tension will increase accordingly as well.

Strumming of cables is caused by the proximity of a preferred vortex formation frequency ω_S to the natural frequency of the structure ω_n. This latter frequency can be obtained as a zero of the lightly-damped Bessel function solution of the lateral dynamics equation above. The preferred frequency of vortex formation is given by the empirical relation $\omega_S = 2\pi SU/d$, where S is the Strouhal number, about 0.16-0.20 for a large range of Re. Strumming of amplitude $d/2$ or greater can occur for $0.6 < \omega_S/\omega_n < 2.0$. The book by Blevins is a good general reference.

14.4 Vehicle Design

The physical layout of a towed vehicle is amenable to the analysis tools of self-propelled vehicles, with the main exceptions that the towpoint presents a large mean force as well as some disturbances, and that the vehicle can be quite heavy in water. Here are basic guidelines to be considered:

1. The towpoint must be located above the vehicle center of in-water weight, for basic roll and pitch stability.

2. The towpoint should be forward of the aerodynamic center, for towing stability reasons.

3. The combined center of mass (material and added mass) should be longitudinally *between* the towpoint and the aerodynamic center, and nearer the towpoint. This will ensure that high-frequency disturbances do not induce excessive pitching.

4. The towpoint should be longitudinally forward of the center of in-air weight, so that the vehicle enters the water fins first, and self-stabilizes with $U > 0$.

5. The center of buoyancy should be behind the in-water center of weight, so that the vehicle pitches downward at small U, and hence the net lift force is downward, away from the surface.

Meeting all of these criteria simultaneously is no small feat, and the performance of the device is very sensitive to small perturbations in the geometry. For this reason, full-scale experiments are commonly used in the design process.