Massachusetts Institute of Technology
Department of Mechanical Engineering

2.160 Identification, Estimation, and Learning

Spring 2006

Problem Set No. 5

Out: March 15, 2006   Due: March 22, 2006

Problem 1

Consider a linear stable system, as shown below. Variable $u(t)$ is input, $y(t)$ is output, and $e(t)$ is uncorrelated white noise with zero mean values. Answer the following questions.

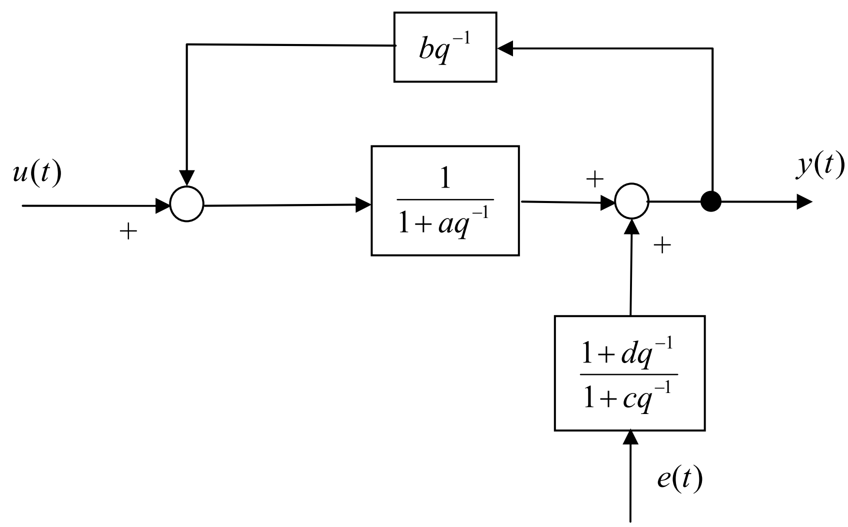

Figure Block diagram

a). Obtain a one-step-ahead predictor for the output: $\hat{y}(t\,|\,t-1)$.

b). Prove that the prediction error $y(t)-\hat{y}(t\,|\,t-1)$ is equivalent to the white noise $e(t)$, if the parameters involved in the system are exactly known.

Problem 2

Consider the continuous-time, Laguerre series expansion of the following two transfer functions:

$$G_1(s) = \frac{1}{(s+1)^2}, \quad G_2(s) = \frac{1}{(s+1)(s+2)}$$

a). The Laguerre series expansion is associated with the following transformation of variables:

$$z = \frac{s+a}{s-a}$$

Change the variable of the above transfer function $G_1(s)$ from *s* to *z*, and obtain the new transfer function $\overline{G}_1(z)$. Find the poles of $\overline{G}_1(z)$, when the parameter *a* is 1; $a = 1$.

b). Setting the parameter *a* to $a = 1$, obtain the coefficients of the Laguerre series expansion of $G_1(s)$:

$$G_1(s) = \sum_{k=1}^{\infty} g_k \frac{\sqrt{2a}}{s+a}\left(\frac{s-a}{s+a}\right)^{k-1}$$

Is the coefficient series $\{g_k\}$ a finite series or an infinite series? What if the parameter *a* is not 1: $a \neq 1$? Explain why.

c). Change the variable of $G_2(s)$ from *s* to *z* in the same way as part a), and obtain the new transfer function $\overline{G}_2(z)$. Find all the poles of the new transfer function $\overline{G}_2(z)$.

d). Set $a = 1.5$, and plot the poles of $\overline{G}_2(z)$ on a complex plane. Next, set $a = 5$, and plot the poles once again. Which parameter value gives faster convergence in the Laguerre series expansion, $a = 1.5$ or $a = 5$? Explain mathematically why it converges more quickly than the other.

Problem 3

Figure 1 shows the cardiovascular network of a human. A high pressure blood flow generated at the left ventricle is distributed through the arterial network. One of the clinically important measures of cardiac function is Cardiac Output, determined from the aortic blood flow, *u*(*t*), as shown in Figure 3. Currently Cardiac Output is directly measured with a catheter inserted into the heart. Since catheterization is a dangerous and costly procedure, it is desirable to estimate the cardiac output from a peripheral pressure using a non-invasive sensor, as shown in Figure 2. This requires the identification of the arterial dynamics relating the peripheral sensor output *y*(*t*) to the input, i.e. the cardiac output, *u*(*t*). Inverting this transfer function *H*(*q*) yields the cardiac output, $u(t) = H^{-1}(q)\, y(t)$.

Non-invasive estimate of cardiac output $u(t)$ using a peripheral pressure sensor $y(t)$:

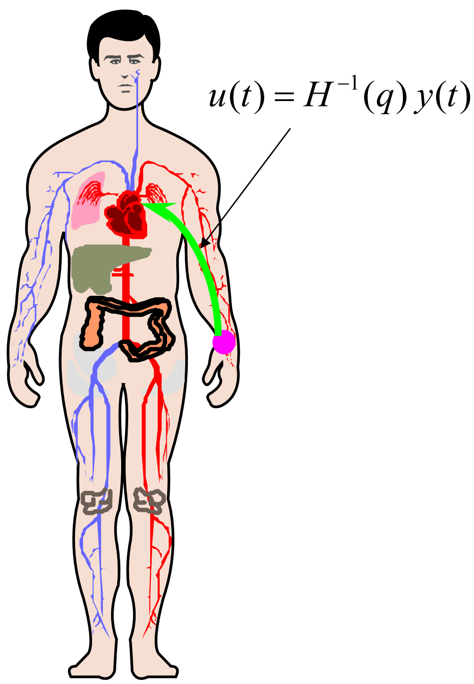

Fig. 1 Cardiovascular network

Figure by MIT OCW.

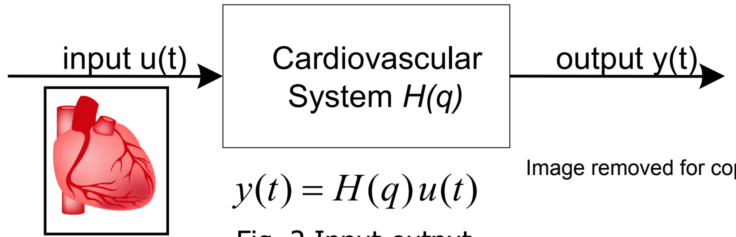

Figure by MIT OCW.

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$$y(t) = H(q)\,u(t)$$

Fig. 2 Input-output

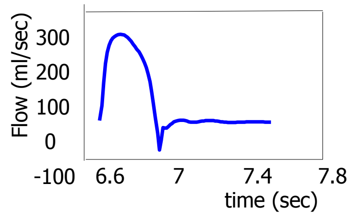

Fig. 3 Cardiac output u(t), aortic flow

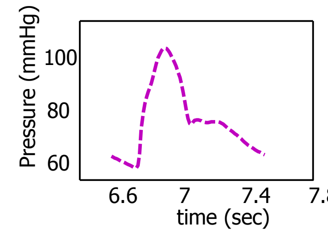

Fig. 4 Radial pressure $y(t)$

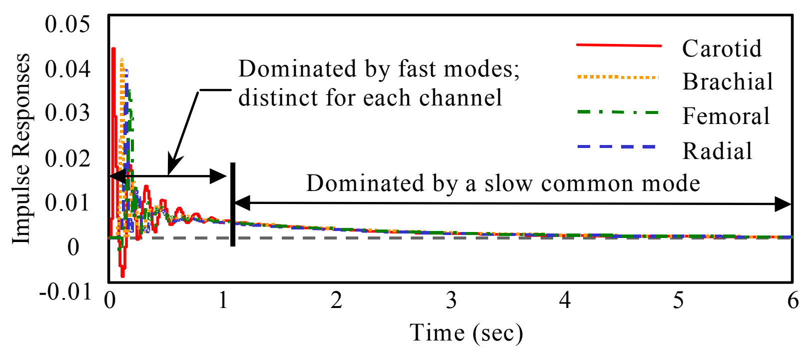

Fig. 5 Impulse response: Aortic flow input to peripheral pressure outputs

Based on experiment data $\{u(t), y(t); t = 1, 2, \cdots\}$, this transfer function can be determined. A FIR model has a unique feature, since it provides a consistent estimate with a standard LSE algorithm although the noise is colored. Unfortunately, however, the impulse response of the arterial dynamics includes a slow decaying mode. Figure 5 shows typical impulse responses from the aortic flow to pressure outputs measured at the carotid, brachial, femoral, and radial arteries. All of the impulse responses show a slow decaying mode. Therefore, the FIR model of every arterial dynamics tends to be of high order, having many parameters to estimate. Answer the following questions.

a). The input data, $u(t)$, are repetitive waveforms, as shown in Figure 3. Therefore the "richness" of the input signal may be limited. Down load an input data file from assignments section, and examine the eigenvalues of the matrix, $\Phi\Phi^T$, where

$$\Phi^T = \begin{pmatrix} u(m) & u(m-1) & \cdots & u(1) \\ u(m+1) & u(m) & \cdots & u(2) \\ \vdots & \vdots & \ddots & \vdots \\ u(m+N-1) & \cdots & \cdots & u(N) \end{pmatrix} \in R^{Nxm}$$

Note that m is the order of the FIR model and that N is the number of usable data sets for identification. Discuss the results with respect to the number of non-zero eigenvalues (including almost zero eigenvalues), the rank of the matrix, and the number of FIR model parameters that can be identified with the input data.

b). Use the Laguerre series expansion for compressing the input data and obtain a compact FIR model. You need to find a proper combination of Laguerre pole a and the reduced FIR model order n through trial-and-error LSE computation. Discuss the results with respect to the speed of conversion, i.e. the reduced FIR model order, v.s. the Laguerre pole as well as the accuracy of the reduced FIR model.