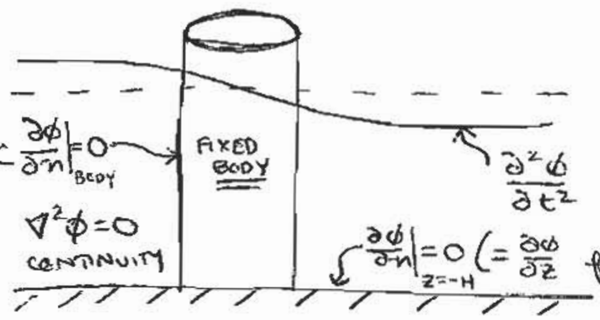
1) a)

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \quad \text{F.S.C (Free Surface Condition)}$$

$$\left.\frac{\partial\phi}{\partial n}\right|_{BODY} = V_B \cdot \hat{n} \quad \text{B.B.C. (Body Boundary Condition)}$$

$\nabla^2\phi = 0$ CONTINUITY

$$\left.\frac{\partial\phi}{\partial n}\right|_{z=-H} = 0 \quad \left[= \frac{\partial\phi}{\partial z} \text{ for a horizontal bottom}\right] \quad \text{B.B.C. (Bottom Boundary Condition)}$$



B.B.C $\left.\frac{\partial\phi}{\partial n}\right|_{BODY} = 0$

$\nabla^2\phi = 0$ CONTINUITY

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \quad \text{F.S.C.}$$

$$\left.\frac{\partial\phi}{\partial n}\right|_{z=-H} = 0 \left(= \frac{\partial\phi}{\partial z} \text{ for a horizontal bottom}\right) \quad \text{B.B.C.}$$

b) FOR THE FLOATING BODY:

$$\phi_T = \phi_I + \phi_D + \phi_R$$

WHERE $\phi_T \equiv$ TOTAL POTENTIAL

$\phi_I \equiv$ INCIDENT WAVE POTENTIAL, DEPENDS ONLY ON THE INCIDENT WAVES

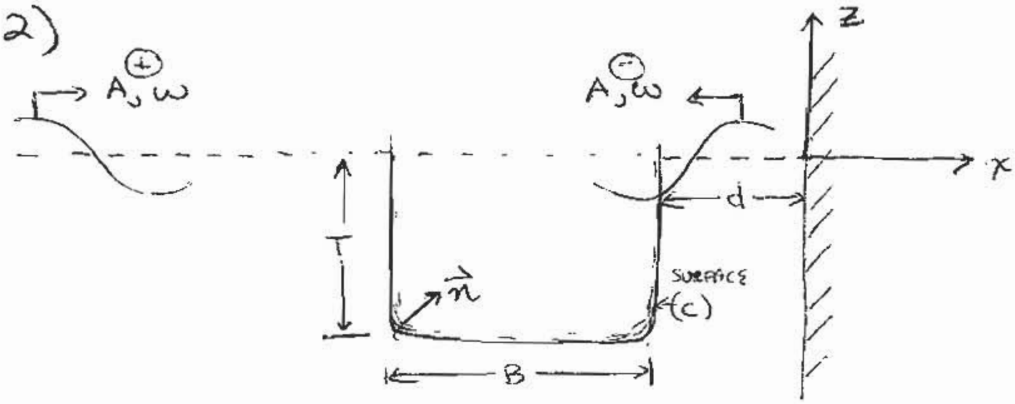
$\phi_D \equiv$ DIFFRACTION POTENTIAL, DEPENDS ON HOW THE INCIDENT WAVES ARE DIFFRACTED BY THE BODY

$\phi_R \equiv$ RADIATION POTENTIAL, DEPENDS ON WAVES CREATED BY MOVEMENT OF THE BODY

FOR THE FIXED BODY:

$$\phi_T = \phi_I + \phi_D \quad , \phi_R = 0$$

PROBLEM 2)



a) In order to derive the F-K exciting forces, you need to define the "ambient wave" on the body. Due to the presence of the pier, this "ambient wave" is the standing wave created by the wall. I have defined the axes so that the wall is located at $x=0$. The velocity potential is that of two waves of equal amplitude (A) that propagate in opposite directions:

$$\phi_I = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz - ikx + i\omega t} + \frac{igA}{\omega} e^{kz + ikx + i\omega t} \right\}$$

[You could verify here that $\left.\frac{\partial \phi_I}{\partial x}\right|_{x=0} = 0$, which is the fixed body boundary cond.]

So THE F-K EXCITING FORCES IN HEAVE AND SWAY CAN BE DESCRIBED BY THE FOLLOWING EXPRESSION

$$F_i = \int_C p\, n_i \, ds$$ where $n_2 \equiv$ SWAY

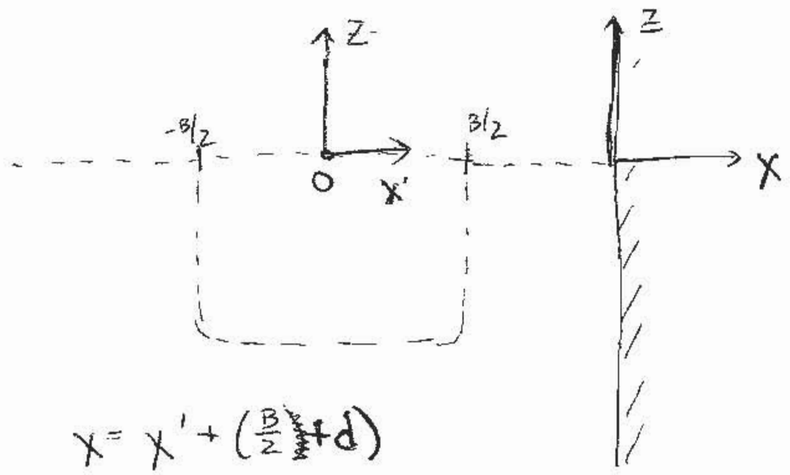
and $n_3 \equiv$ HEAVE

and $p = -\rho \frac{\partial \phi_I}{\partial t}$

HW7 SOLUTIONS SPRING 2005

PROBLEM 2 CONT.)

b)



{ NOTE THAT HERE "X" DIRECTION IS SWAY }

$$x = x' + \left(\frac{B}{2} + d\right)$$

DEFINE ANOTHER SET OF AXES AS PICTURED ABOVE, WITH THE ORIGIN CENTERED ON THE 2D BARGE SECTION

THE INCIDENT WAVE POTENTIAL BECOMES:

$$\phi_I = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz} e^{-ik(x'+\frac{B}{2}+d)+i\omega t} + \frac{igA}{\omega} e^{kz} e^{ik(x'+\frac{B}{2}+d)+i\omega t} \right\}$$

$$= \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz} \left[e^{-ikx' - ik(\frac{B}{2}+d)} + e^{ikx' + ik(\frac{B}{2}+d)} \right] e^{i\omega t} \right\}$$

HEAVE: $\quad F_3 = \int_{-B/2}^{B/2} p_I \, dx' \quad$ where $p_I = -\rho \frac{\partial \phi_I}{\partial t}$

USE TRIGONOMETRIC IDENTITY: $e^{-i\alpha} + e^{i\alpha} = 2\cos\alpha$

let $\alpha = kx' + k(\frac{B}{2}+d)$

$$\therefore e^{-i k[x' + \frac{B}{2} + d]} + e^{ik[x' + \frac{B}{2}+d]} = 2\cos\left(kx' + k\left(\frac{B}{2}+d\right)\right)$$

ALSO USE TRIG IDENTITY: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$\therefore 2\cos\left(kx' + k\left(\frac{B}{2}+d\right)\right) = 2\cos(kx')\cos\left(k\left(\frac{B}{2}+d\right)\right) - 2\sin(kx')\sin\left(k\left(\frac{B}{2}+\right.\right.$$

$$\therefore \phi_I = \mathrm{Re}\left\{ \frac{igA}{\omega} e^{kz} \, 2\left[\cos(kx')\cos\left(k\left(\frac{B}{2}+d\right)\right) - \sin(kx')\sin\left(k\left(\frac{B}{2}+d\right)\right)\right] e^{i\omega t}\right.$$

PROBLEM 2 CONT.)

b CONT.)

$$p_I = -\rho \frac{\partial \phi_I}{\partial t} = -\rho\, \mathrm{Re}\left\{ \frac{igA}{\omega} \cdot i\omega \cdot e^{kz} \cdot 2\left[\cos(kx')\cos(k\tfrac{B}{2}+kd) - \sin(kx')\sin(k\tfrac{B}{2}+kd)\right] \cdot e^{i\omega t} \right\}$$

NOTE: Because I am integrating wrt x' over the interval $-\frac{B}{2}$ to $\frac{B}{2}$, the $\sin(kx')$ term drops out $\therefore$

$$F_3 = \int_{-B/2}^{B/2} \rho\, \mathrm{Re}\left\{2gA\, e^{kz}\cos(kx')\cos(k(\tfrac{B}{2}+d))e^{i\omega t}\right\} dx'$$

$$= 2\rho g A e^{k(-T)} \cos(k(\tfrac{B}{2}+d)) \cos \omega t \underbrace{\int_{-B/2}^{B/2} \cos(kx')\, dx'}_{= \frac{2}{k}\sin\frac{kB}{2}}$$

$$F_3 = \frac{4\rho g A}{k} e^{-kT} \cos\omega t \cos(k(\tfrac{B}{2}+d)) \sin\frac{kB}{2}$$

SO THIS IS THE HEAVE F-K FORCE. IT WILL VANISH WHEN EITHER $\cos(k(\frac{B}{2}+d)) = 0$ OR $\sin\frac{kB}{2} = 0$.

FOR $\cos(k(\frac{B}{2}+d)) = 0$, $k(\frac{B}{2}+d) = \frac{\pi}{2}(2n+1)$ for $n = \pm 1, 2, \ldots$

$$k = \frac{\frac{\pi}{2}(2n+1)}{\frac{1}{2}(B+2d)} = \frac{\pi(2n+1)}{B+2d} \text{ and } \lambda = \frac{2\pi}{k} = \frac{2\pi(B+2d)}{\pi(2n+1)} = \frac{2(B+2d)}{2n+1}$$

FOR $\sin\frac{kB}{2} = 0$, $\frac{kB}{2} = m\pi$ for $m = \pm 1, 2, \ldots$

$$k = \frac{2m\pi}{B} \text{ and } \lambda = \frac{2\pi}{k} = \frac{2\pi B}{2m\pi} = \frac{B}{m}$$

SO THE HEAVE F-K FORCE $\lambda = \frac{2(B+2d)}{\ }$ OR $\lambda = \frac{B}{\ }$ for $n = \pm 1, 2, \ldots$

PROBLEM 2b CONT.)

SWAY: THE SWAY F-K EXCITING FORCE DEPENDS ONLY ON THE PRESSURE DISTRIBUTION ON THE TWO VERTICAL SIDES

GOING THROUGH A SIMILAR PROCESS AS THE HEAVE SOLUTION ABOVE, YOU FIND THAT:

$$F_2 = \rho g A \,\mathrm{Re}\left\{ \int_{-T}^{0} dz\, e^{kz} (+i) \sin\frac{kB}{2} \left[e^{-ik(\frac{B}{2}+d)} - e^{ik(\frac{B}{2}+d)} \right] e^{i\omega t} \right\}$$

$$= \rho g A \,\mathrm{Re}\left\{ \int_{-T}^{0} dz\, e^{kz} (i) \sin\frac{kB}{2} (-2i) \sin\left(k\left(\tfrac{B}{2}+d\right)\right) e^{i\omega t} \right\}$$

$\therefore$ THE SWAY F-K EXCITING FORCE WILL DISAPPEAR WHEN:

$$\sin\frac{kB}{2} = 0 \quad \text{OR} \quad \sin\left(k\left(\tfrac{B}{2}+d\right)\right) = 0.$$

FOR $\sin\frac{kB}{2} = 0$, $\frac{kB}{2} = n\pi$ for $n = \pm 1, 2, \ldots$

$$k = \frac{2n\pi}{B} \quad \text{and} \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{2n\pi}{B}} = \frac{B}{n}$$

FOR $\sin\left(k\left(\frac{B}{2}+d\right)\right) = 0$, $k\left(\frac{B}{2}+d\right) = m\pi$ for $m = \pm 1, 2, \ldots$

$$k = \frac{m\pi}{\left(\frac{B}{2}+d\right)} \quad \text{and} \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{m\pi}{\left(\frac{B}{2}+d\right)}} =$$

$$\lambda = \frac{2\left(\frac{B}{2}+d\right)}{m} = \frac{B+2d}{m}$$

SO THE SWAY F-K VANISHES AT

$$\lambda = \frac{B}{n} \quad \text{or} \quad \lambda = \frac{B+2d}{n} \quad \text{for } n = \pm 1, 2, \ldots$$