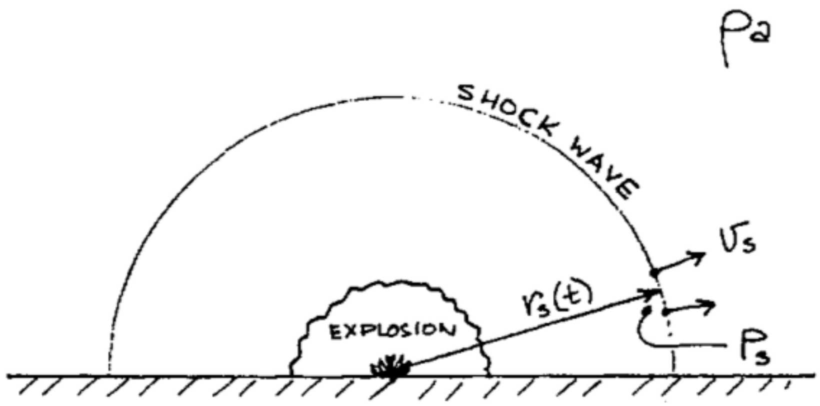
MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 7.09
This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A strong explosion (like an atomic bomb) causes a spherically symmetric shock wave to move through the air radially out from the origin. As the shock sweeps by, it causes a sudden rise in the pressure and sets the initially static air into radially outward motion.

It can be argued from strong shock wave theory that if the undisturbed atmosphere is homogeneous at a density ρ_a, the velocity v_s of the shock, as well as the pressure p_s and the wind speed just behind the shock wave, should depend only on the density ρ_a, the total distance r_s of the shock wave from the origin, and the total energy E released by the explosion.

(a) Show that:

$$v_s = \text{const.} \left(\frac{E}{\rho_a}\right)^{\frac{1}{2}} \cdot r_s^{-\frac{3}{2}} \tag{7.09a}$$

$$p_s = \text{const. } E \cdot r_s^{-3} \tag{7.09b}$$

(b) Obtain an expression for the shock's radial position as a function of time (the expression may involve one unknown dimensionless constant). Show how the strengths of two different bomb explosions, as measured by their energy releases, can be compared based on film information about their shock wave positions as a function of time.

Solution:

(a) We have to find $v_s = f(E, \rho_a, r_s)$. Hence, let us assume on dimensional grounds

$$v_s = \text{const. } E^a \rho_a^b r_s^c \tag{7.09c}$$

Equivalently, this result can be written in terms of $\left[LT^{-1}\right] = \left[ML^2T^{-1}\right]^a \left[ML^{-3}\right]^b L^c$. Solving for each exponent, we find that $a = 0.5$, $b = -0.5$ and $c = -1.5$, and accordingly,

$$v_s = \text{const.} \left(\frac{E}{\rho_a}\right)^{\frac{1}{2}} \cdot r_s^{-\frac{3}{2}} \tag{7.09d}$$

Also, we wish to find $p_s = g(E, \rho_a, r_s)$, where we have assumed, for the moment, that p_s can depend on ρ_a. Again,

$$p_s = \text{const. } E^a \rho_a^b r_s^c \tag{7.09e}$$

This result gives $\left[ML^{-1}T^{-2}\right] = \left[ML^2T^{-1}\right]^a \left[ML^{-3}\right]^b L^c$. For this result to be valid, $a = 1$, $b = 0$ and $c = -3$ and so

$$p_s = \text{const. } E \cdot r_s^{-3} \tag{7.09f}$$

The dependence on ρ_a drops out by itself!

(b) From our analysis from part (a), we have

$$v_s = C \left(\frac{E}{\rho_a}\right)^{\frac{1}{2}} \cdot r_s^{-\frac{3}{2}} \tag{7.09g}$$

where we let C be a constant. It follow then, that

$$v_s = \frac{dr_s}{dt} = C \left(\frac{E}{\rho_a}\right)^{\frac{1}{2}} \cdot r_s^{-\frac{3}{2}} \tag{7.09h}$$

Integrating this result with the initial condition $r = 0$ at $t = 0$, we have

$$\frac{2}{5} r^{\frac{5}{2}} = C \left(\frac{E}{\rho_a}\right)^{\frac{1}{2}} t \tag{7.09i}$$

and hence $E \propto r^5$ at any instant t. The above expression indicates that any instant t, the energy of the explosion is proportional to the 5^{th} power of the radial extent r_s of the corresponding shock wave.

□

Problem Solution by Mayank Kumar, Thomas Ober (Updated), Fall 2007

