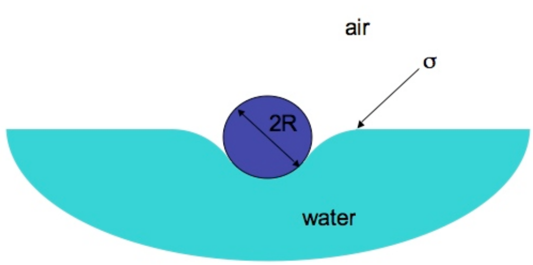
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 2.6

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



Cross section of a hair (Left), and water strider photograph (Right).

The water strider, or pond skater, is a slender insect, about $\frac{1}{2}$ cm long, that runs or 'skates' over the surfaces of ponds and streams. It stays easily on the surface because its feet (tarsi) are equipped with numerous fine, non-wetting hairs.

Suppose we model one of these hairs as a long cylinder of radius R [1] made of completely non-wetting material (contact angle 180 degrees), and assume that it is set down on the water with its axis parallel to the surface, as sketched. The surface tension is $0.07[N/m]$ (water/hair).

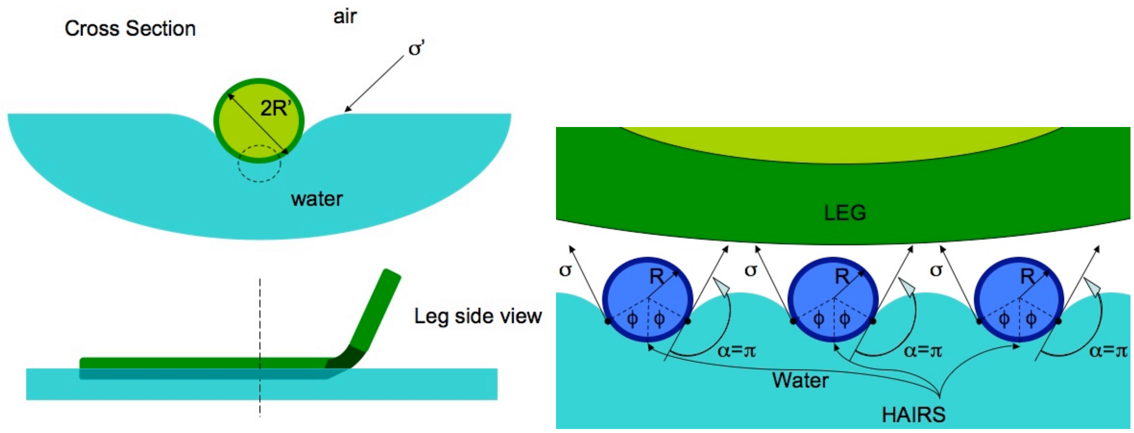
- (a) Show that, as the cylinder, or hair, is brought into contact with the water and then depressed into it, the lift force exerted on it by surface tension first increases, then reaches a maximum at a certain depression and finally decreases as the cylinder is depressed further. What is the maximum value of the surface-tension-induced lift force per unit cylinder length?
- (b) What is the criterion for the gravitational effects to have a negligible effect on the (maximum) total lift force? Is it likely that this criterion is satisfied for the pond skater's tarsi?
- (c) If a pond skater weights 0.05 grams (note that this is only a guess, not a figure based on observations of the real insect) [2] , what minimum local length of hair must it have on its feet to keep it on top of the water?
- (d) What is the shape of the water surface around the leg when the force is maximum? (contact angle, between the water and the leg, 180 degrees [3]) Idealize the leg as a perfect cylinder (no hairs) and as before assume the case that produces maximum force. (Use R' for the leg radius.)

[1] Roughly 2.5 microns, see added preamble

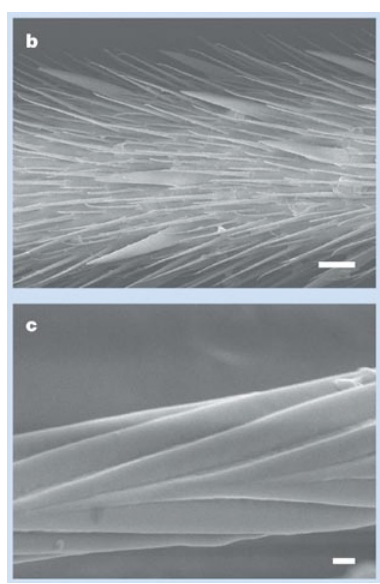
[2] Actually, accordingly to the Wikipedia the weight of these insects is 0.01 grams, not a bad guess.

[3] Accordingly to the Wikipedia the effective angle for the legs (due to the properties of the hairs) is roughly 170 degrees, superhydrophobic, so this is not a bad approximation.

Preamble:



If we make a diagram of the leg, we realize that this is a highly idealized problem; the position of the hairs relative to the leg and the presence of hairs next to each other will change the force that different hairs provide to the leg. Also, if we look at an electron microscope photograph of the hairs, we realize that their shape is not circular and not constant. Nevertheless,we are not interested in an exact solution, but we want an estimate of the order of magnitude of the forces involved, and the geometric restrictions. Then, we'll consider a single cylindrical hair isolated on the water surface as shown in the problem statement, let's just keep in mind that we are just calculating approximate values for something as complex as the leg structure shown in the image.



Scanning electron microscope images of a leg showing numerous oriented spindly microsetae (b) and the fine nanoscale grooved structures on a seta (c). Scale bars: b, 20 μm; c, 200 nm. Taken from 'Water-repellent legs of water striders' by Xuefeng Gao, and Lei Jiang, Nature 432, 36 (4 November 2004).