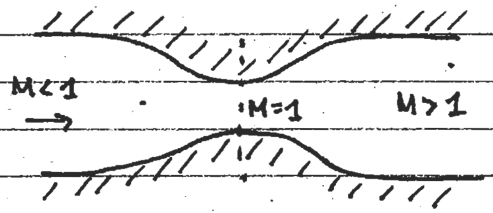
O.H. Wed. 3-4

2.26 - Lect. 3 (H.W. Due Friday - @ 5:00?)

Compressible 1D duct flow w. friction

Recall from last time



Isentropic flow:

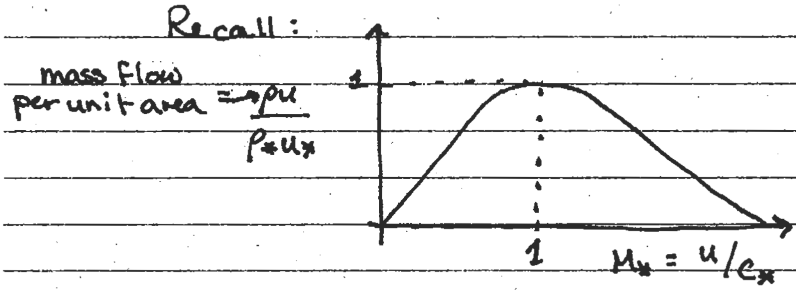
$$\left(\frac{\rho_0}{\rho}\right)^{\gamma-1} = \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

choose one of these as ref. state: subscript o = stagnation condition; superscript * = sonic condition

$$\frac{P}{P_0} = \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{-\frac{\gamma}{\gamma-1}}$$

$$\frac{A}{A_*} = \frac{1}{M}\left(\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Recall:



what is M_* ?

$$M_* = \underbrace{\frac{u}{c_*}}_{\text{constant for a given stagnation condition}} = \frac{u}{c}\frac{c}{c_*}\underbrace{\frac{c_*}{c_0}}_{\sqrt{\frac{2}{\gamma+1}}} = M\frac{c}{c_0}\frac{c_0}{c_*} = M\left(\frac{T}{T_0}\right)^{1/2}\left(\frac{\gamma+1}{2}\right)^{1/2}$$

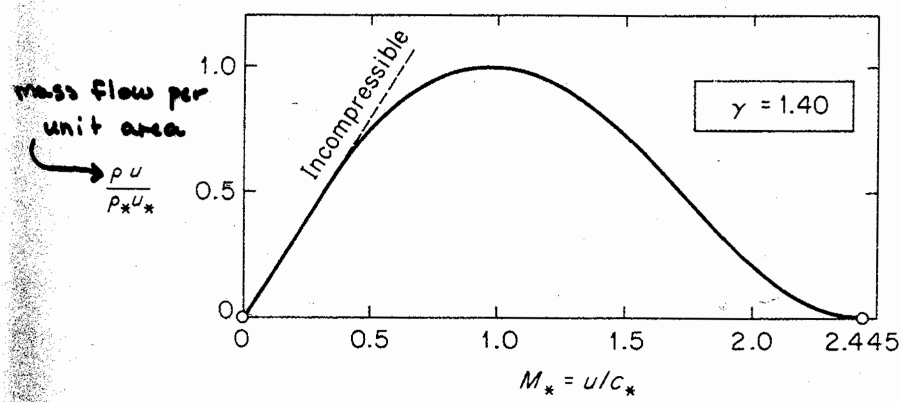


Figure 5.18
Mass flux as a function of M_*, for a perfect gas with $\gamma = 1.40$.

In terms of M_*, the normalized mass flux becomes, from (5.62),

$$\frac{\rho u}{\rho_* u_*} = M_* \left(\frac{\gamma + 1}{2} - \frac{\gamma - 1}{2} M_*^2 \right)^{1/(\gamma - 1)} \tag{5.65}$$

which is shown in Fig. 5.18. This illustrates the maximum at $M = M_* = 1$. For low Mach numbers, the curve rises as a straight line, corresponding to the linear increase in mass flux with speed for incom- ... $[M_* = \sqrt{(\gamma + 1)/(\gamma - 1)}]$ the mass flux is

$$M_* = M\left(\frac{\gamma+1}{2}\right)^{1/2}\left[\frac{1}{1+\frac{\gamma-1}{2}M^2}\right]^{1/2} = \frac{M}{\left[\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M^2\right]^{1/2}}$$

As $M \to \infty$ (i.e. $c \to 0$)

$$M_* \to \frac{\cancel{M}}{\left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}\cancel{M}} = \left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} = 2.449\ldots$$

for $\gamma = 1.4$

this is where mass flux $\to 0$ in the plot!

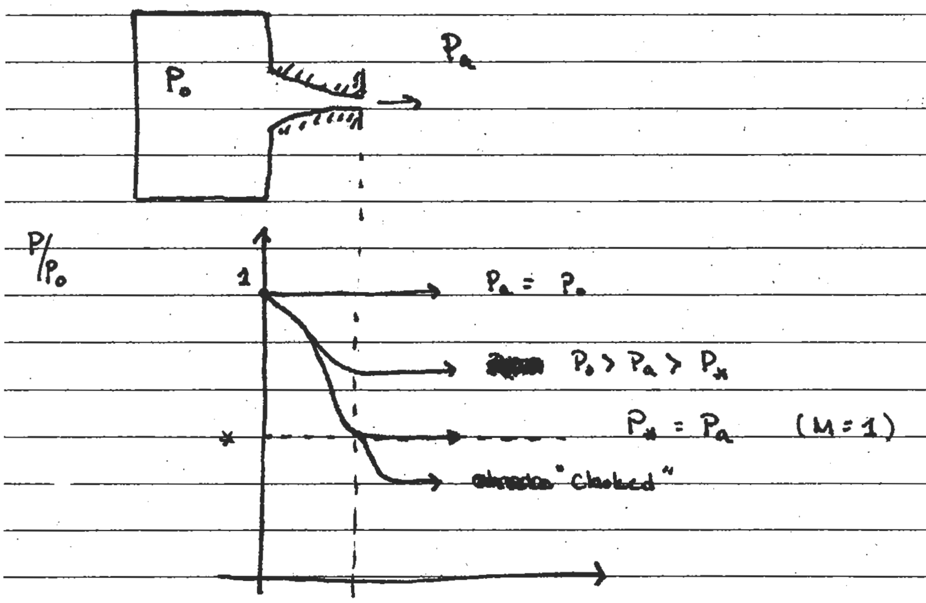
Interpretation: @ $M_* = 2.449\ldots \Rightarrow M \to \infty \Rightarrow c \to 0$

$\Rightarrow \boxed{\rho \to 0}$

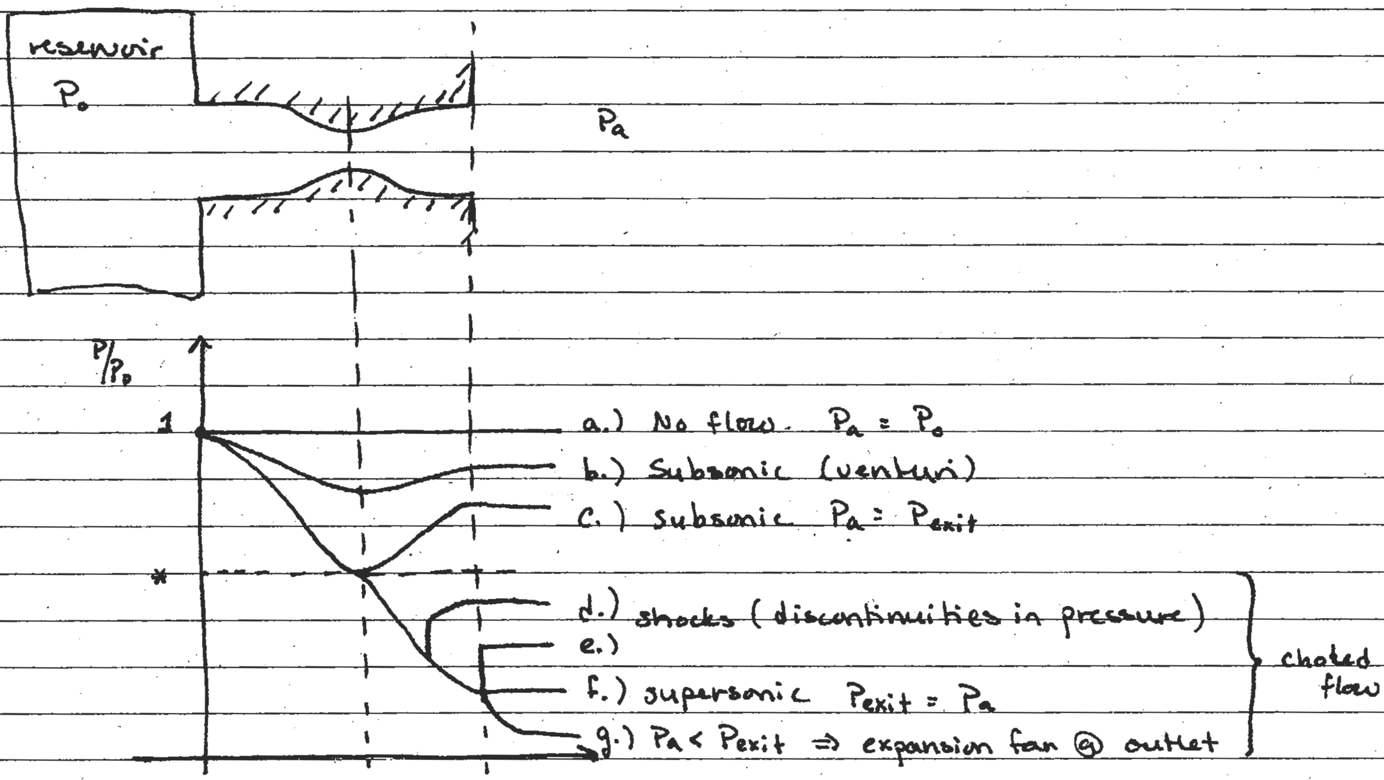
~~Mass flux~~ Density is going to zero FASTER than the velocity is increasing.

$\Rightarrow$ mass flux $\to$ zero.

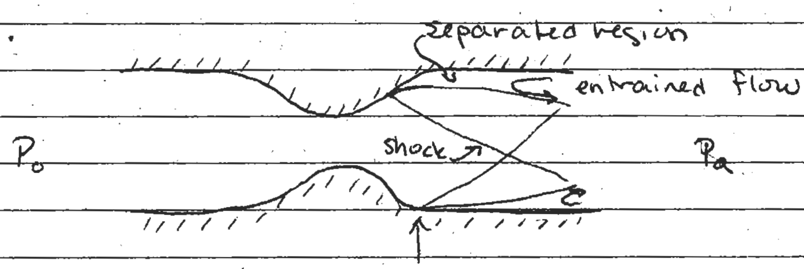
<u>Converging nozzle</u> <u>(from last time)</u>:



Converging / Diverging Nozzle



- Change mass flux by changing P_a
- For d-g, mass flux is exactly the same (lowering pressure doesn't → more mass flux)
- "Real" supersonic nozzles, flow separates



Separation pt.

roughly: $P_s \sim 0.4\, P_a$
(empirical)

Effects of viscosity

$$\delta_{ik} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$$

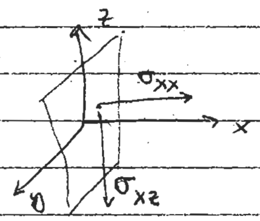
Stress tensor for a viscous fluid:

$$\sigma_{ik} = -P\delta_{ik} + \underbrace{2\mu}_{\text{shear viscosity}}\overbrace{\left(D_{ik} - \underbrace{\tfrac{1}{3}\delta_{ik} D_{mm}\right) + \underbrace{\mu_b}_{\text{bulk viscosity}}\delta_{ik} \overbrace{D_{mm}}^{\nabla\cdot u}}_{\text{expansion stress}}}^{\text{shear stress}}$$

$\equiv -P\delta_{ik} + d_{ij}$

= force on a plane normal to the i direction in the k direction

e.g. σ_{xx}, σ_{xz}



(deformation tensor)

where $D_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)$.

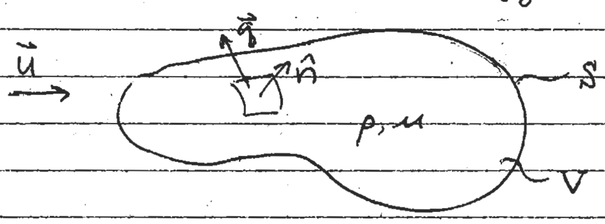
(Note: for an incompressible fluid, this reduces to

$$\sigma_{ik} = -P\delta_{ik} + \mu\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)$$

Conservation of momentum:

$$\boxed{\rho\frac{D\bar{u}}{Dt} = \rho\bar{g} + \nabla\cdot\underline{\underline{\sigma}}} \Rightarrow \rho g_i + \frac{\partial}{\partial x_j}\sigma_{ji}\cdot \underbrace{\hat{e}_i}_{\text{unit vector}} = \rho\frac{Du_i}{Dt}$$

Conservation of energy:



internal energy, kinetic energy

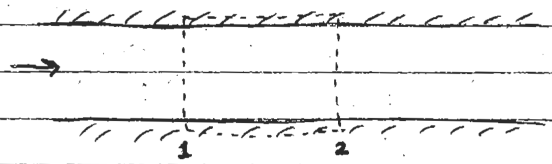
$$\underbrace{\frac{d}{dt}\int_V \rho\left(e+\tfrac{1}{2}u^2\right)dV}_{\text{time rate of change of total energy}} = \underbrace{-\int_S \rho\left(e+\tfrac{1}{2}u^2\right)\bar{u}\cdot\hat{n}\,dA}_{\text{outflow of energy}}$$

surface force/unit area "traction"

$$+\underbrace{\int_V \rho\,\bar{g}\cdot\bar{u}\,dV}_{\text{change in P.E./time}} \underbrace{-\int_S \bar{q}\cdot\hat{n}\,dA}_{\text{heat out}} + \underbrace{\int_S \bar{T}\cdot\bar{u}\,dA}_{\text{work against pressure + work by viscosity}}$$

$T_k(\hat{n}) = n_i \sigma_{ik}$

Consider a "1D" duct; const cross-section; steady state; adiabatic; neglect P.E.



Cons. of energy becomes:

$$0 = -\int_S \rho\left(e+\tfrac{1}{2}u^2\right)\bar{u}\cdot\hat{n}\,dA + \int_S \bar{T}\cdot\bar{u}\,dA$$

$\frac{ds}{dx} > 0$

Assume friction generates entropy (irreversibilities) but does not significantly change the energy.

$$\rho\left(e+\tfrac{1}{2}u^2\right)u + pu\Big|_1 = \rho\left(e+\tfrac{1}{2}u^2\right)u + pu\Big|_2$$

$$\underbrace{(\rho u \cancel{A})_1}_{\dot{m}}\left(h+\tfrac{1}{2}u^2\right)\Big|_1 = \underbrace{(\rho u \cancel{A})_2}_{\dot{m}}\left(h+\tfrac{1}{2}u^2\right)\Big|_2$$

$$\boxed{h_1 + \tfrac{1}{2}u_1^2 = h_2 + \tfrac{1}{2}u_2^2 = \text{const}}$$

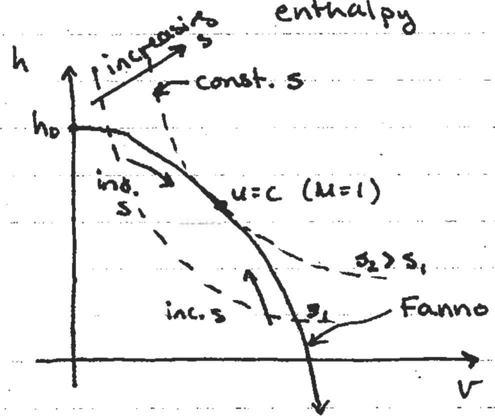
Cons. of mass

mass flux $= J =$ const. $= \rho u$ (recall A is constant)

$$\Rightarrow u = J/\rho = J v$$

Combine w. cons. of energy

$$\underbrace{h_0}_{\text{stagnation enthalpy}} = h + \tfrac{1}{2} J^2 v^2 \Rightarrow \boxed{h = h_0 - \tfrac{1}{2} J^2 v^2}$$



(As fluid moves down the pipe $\Rightarrow$ move along the Fanno line)

Lines of constant entropy:

$$\underbrace{\Delta s}_{\text{isentropic } = 0} = C_v \ln(T/T_0) + R \ln(v/v_0) \qquad \text{(ideal gas)}$$

$$\Delta h = C_p \Delta T \qquad \text{(perfect gas)}$$

$$0 = C_v \ln(h/h_0) + R \ln(v/v_0)$$

$$\Rightarrow h = h_0 (v/v_0)^{1-\gamma} \Rightarrow \underline{\text{concave up}}$$

(true in general)

Point where entropy is maximized is where the line of const s is tangent to the Fanno line

$$\left(\frac{\partial h}{\partial v}\right)_s = \left(\frac{\partial h}{\partial v}\right)_{Fanno}$$

$$\left(\frac{\partial h}{\partial v}\right)_s = \underbrace{\left(\frac{\partial h}{\partial P}\right)_s}_{v} \underbrace{\left(\frac{\partial P}{\partial v}\right)_s}_{\left(\frac{\partial P}{\partial \rho}\right)_s \left(\frac{\partial \rho}{\partial v}\right)_s} \qquad dh = Tds + vdP$$

$$\left(\frac{\partial P}{\partial v}\right)_s = \underbrace{\left(\frac{\partial P}{\partial \rho}\right)_s}_{c^2} \underbrace{\left(\frac{\partial \rho}{\partial v}\right)_s}_{-1/v^2}$$

$$= -\frac{c^2}{v}$$

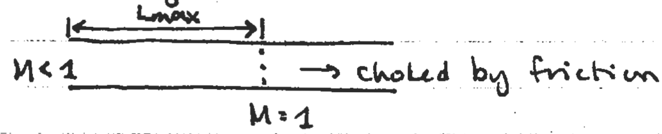
$$\left(\frac{\partial h}{\partial v}\right)_{Fanno} = -J^2 v = -(\rho u^2) v = -\frac{u^2}{v}$$

$$\Rightarrow \frac{c^2}{v} = \frac{u^2}{v} \Rightarrow \quad u = c \quad \text{sonic condition!}$$

$\therefore$ Friction drives the Mach number towards unity

- an initially subsonic flow can only be ~~ac~~ accelerated to $M = 1$ ("frictional choking")
- an initially supersonic flow can only decelerate past $M = 1$ if it jumps discontinuously to a new fanno line (shock).

For a given ΔP



<u>Friction factor in Compressible flows</u>

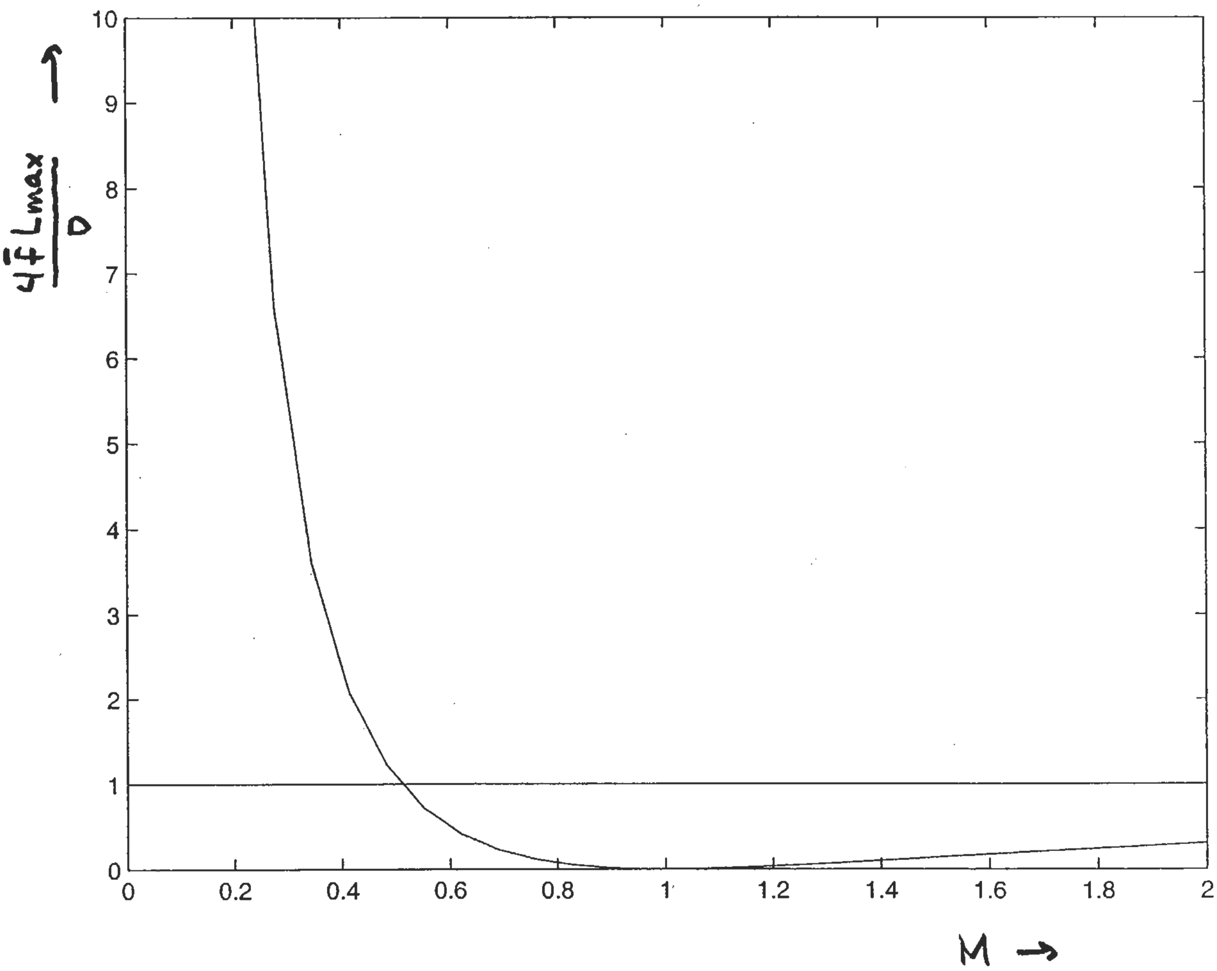
Recall for <u>incompressible</u> pipe flow:

$$\frac{f}{4} = \frac{\tau_w}{\frac{1}{2}\rho u^2} \qquad \text{Darcy friction factor (Moody chart)}$$

$$\left(f = \frac{64}{Re_{D_H}} \text{ for laminar flow}\right)$$

$$D_H = \frac{4A_c}{P}$$

P ↑ wetted perimeter



$\frac{4\bar{f}L_{max}}{D}$
10
9
8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
M

NASA fails to fly fastest plane

EDWARDS AIR FORCE BASE, California (CNN) -- The X-43A, a "hypersonic" experimental aircraft attached to a Pegasus booster rocket, failed the first of three planned test flights Saturday.

The flight plan called for the X-43A to separate from the booster rocket and free-fall back to earth in about 10 seconds, landing in the Pacific Ocean. Instead, the aircraft fell directly into the ocean.

The aircraft was launched 95,000 feet above the Pacific Missile Test Range here, from a NASA B-52.

NASA had hoped the aircraft would reach speeds up to seven times the speed of sound, or almost 5,000 miles per hour.

Two other planned test are now on hold, officials said.

The hypersonic combustion engine technology, an elusive engineering goal for decades, could someday allow craft to fly 10 times faster than the speed of sound, a pace now reserved to conventional rockets.

It's called "scramjet" technology, and it relies on air-breathing engines instead of rocket power to achieve its tremendous speeds. Currently, the SR-71 holds the title as the fastest air-breathing plane in the world. It has cruised slightly above Mach 3, or three times the speed of sound. A plane flying at the speed of sound can travel a mile in about five seconds.

Two other tests of the of the $185 million Hyper-X flight research program planned for later this year and late next year, have been put on hold.

MicroCraft, Inc. of Tullahoma, Tenn., is the industry partner chosen by NASA to construct the X-43 vehicles. Orbital Sciences Corporation's Launch Vehicles Division in Chandler, Ariz. will construct the Hyper-X launch vehicles.

The proposed X-43 hypersonic plane would be a marked improvement over a conventional rocket-powered craft, NASA said. The former uses atmospheric oxygen to combust its fuel.

But the latter must carry its own supply of the element, considerably increasing its weight. Hypersonic is defined as traveling at a speed equal to about five times the speed of sound or greater.

The scramjet has a simple mechanical design with no moving parts. Rather than using a rotating compressor like a turbojet engine, the forward movement compresses air into the engine.

Fuel, in this case hydrogen, is injected and the expanding hot gases from combustion accelerate the exhaust air and create thrust.

Scramjet technology could also allow more traditional aircraft to greatly reduce flight times and costs. Practical applications could be possible within decades, according to NASA.

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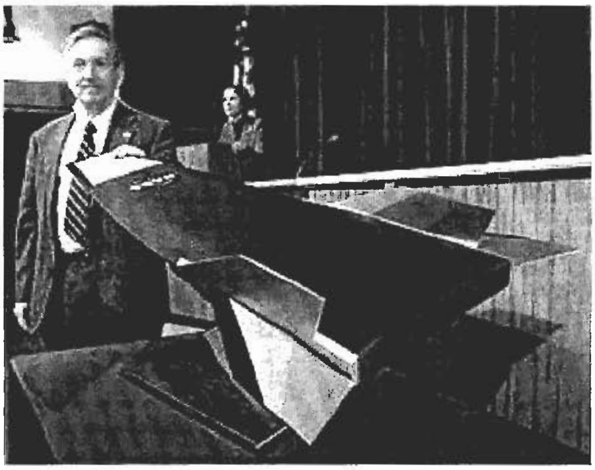

The fastest plane on earth? Vince Rausch, project manager for the NASA X-43A, an experimental aircraft designed to fly up to seven times the speed of sound, poses with a scale model of the aircraft during a press briefing at Edwards Air Force Base, in the Southern California desert. The aircraft will be dropped from a B52 bomber over the Pacific Ocean off the California coast during its maiden flight in mid-May. **Photo: AP/Reed Saxon**

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