

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
CAMBRIDGE, MASSACHUSETTS 02139
2.29 NUMERICAL FLUID MECHANICS — SPRING 2015

Problem Set 6

Issued: Monday, April 27, 2015

Due: Wednesday, May 6, 2015

Grading Note: Please provide your solutions either as hand-written/hard-copy solutions or by submitting via course website. MATLAB[®] codes should be submitted via course website. The bulk of the grades will be given to detailed explanations and to algorithms and numerical schemes that capture the essence of the numerical problems. We know that successful coding of numerical schemes can be time consuming and prone to small errors. Such small errors or omissions in a code will not be heavily penalized.

The goals of this Problem Set are to: (i) utilize and evaluate the need for higher order time-marching schemes for systems of different degrees of nonlinearity; (ii) learn about grid generation methods and algorithms.

Problem 1: A tribute to MIT Prof. Edward Lorenz. The now famous Lorenz equations were published in 1963 (Lorenz, E.N., *Deterministic Nonperiodic Flow*, Journal of Atmospheric Science, 20, 1963, 130-141). Lorenz obtained these equations by first expressing the solution of two-dimensional (x, z) convection equations as Fourier series. He then truncated the series to three terms and obtained the following ODEs for the non-dimensional amplitudes X, Y, Z :

$$\begin{aligned}\frac{dX}{dt} &= -\sigma X + \sigma Y \\ \frac{dY}{dt} &= rX - Y - XZ \\ \frac{dZ}{dt} &= XY - bZ\end{aligned}$$

The classic values of the parameters (used by Lorenz) are: $\sigma = 10$, $b = 8/3$ and $r = 28$ (r is then such that the Rayleigh number is slightly supercritical, i.e. larger than the critical Rayleigh number for instability of steady convection). In these equations, X corresponds to the magnitude of atmospheric fluid motions, and Y and Z to temperature variations in the horizontal and vertical directions, respectively.

a) Starting from the initial conditions $X = Y = Z = 5$, and integrating from $t = 0$ to 60, solve the Lorenz ODEs using methods defined in Lectures on Time-Marching: i) Forward Euler; ii) Heun's; iii) Implicit Euler; iv) Runge-Kutta of 4th order; v) AM3; vi) AM4 and vii) Milne's method. You can start the multistep methods using Runge-Kutta. For each of the methods, plot the results of X versus t and briefly compare these results. (Hints: remember that implicit methods lead to a nonlinear system to be solved at each time step; and that, as mentioned in lecture, Milne's method can become unstable, e.g. see also Chap 26.2.4 of Chapra and Canale).

b) For each of the methods in part a), plot the state-space representation of your solution (i.e. plot the solution in the X, Y, Z plane, e.g. using plot3 in MATLAB). Very briefly compare the results.

c) Repeat the state-space (X, Y, Z) plots of part b) for each of the methods, but for two different values of the parameter r , specifically: $r = 10$ (the Lorenz equations then have two stable equilibrium points) and for $r = 100$ (the Lorenz equations then have an attractor with a single periodic orbit). Very briefly describe the results.

Problem 2: Planning a Trip to a Tropical Island. A good friend of yours, Pedro Lopez, is inviting you for a fun trip to a tropical island after the end of classes. He is working on a flying carpet and is interested in using it to travel to the island (for more information on flying carpets, see Argentina et al, 2008. *Settling and Swimming of Flexible Fluid-Lubricated Foils*, PRL, 224503. <http://www.seas.harvard.edu/softmat/downloads/2007-08.pdf>). To get started, Pedro has been working on an idealized 2D problem. To create a grid over this idealized carpet, he has given you the MATLAB script “flying_carpet_PL.m”, available on course website.

a) Run the MATLAB code. Based on the code and its results:

- i) Which algorithm is Pedro using to grid the idealized carpet?
- ii) Does the grid you obtained seem to be acceptable, i.e. are there locations for which you would like to modify it? If yes, why?

b) Pedro is investigating different shapes for the carpet. He gives you the following definition of the four boundaries of another carpet:

```
f_s=[xsi; -sin(2*pi*xsi)/4];  
f_n=[1+xsi; 3-sin(-2*pi*xsi)/4+2*xsi];  
f_e=[(1+eta); (1+eta).^3-2*(1+eta)+1];  
f_w=[eta; 3*sqrt(eta)];
```

- i) Modify the MATLAB script, run it for this shape and plot the results.
- ii) Is the gridding procedure acceptable for this shape, i.e. is the grid you obtain usable? Why?

c) Assuming that you would like to utilize the techniques seen in class to improve the grids obtained in part a-b), briefly define (in a few words) an algorithm that you could utilize to obtain a more regular grid. (Hint: specify the computational and physical domains, the equations to be solved and how the terms in these equations could be computed). Do not write any code.

d) Utilize the function “delaunay(x,y)” in MATLAB to mesh the flying carpet of part a) using triangles. To plot your mesh, you can use the function “trimesh(tri,x,y)” where the matrix tri is the result of the Delaunay script. Is there a major issue with this mesh? If yes, suggest a solution.

MIT OpenCourseWare
<http://ocw.mit.edu>

2.29 Numerical Fluid Mechanics

Spring 2015

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.