Lecture 6

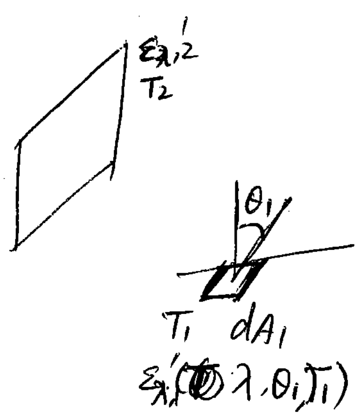
Review of last lecture:

a) Specular surfaces

b) Windows, semitransparent

c) spectral dependent

d) Monte Carlo method



From $\varepsilon'_{\lambda,1} \rightarrow \varepsilon_1(T_1)$

$$dQ_{e1} = \varepsilon_1 \sigma T_1^4 \, dA_1$$

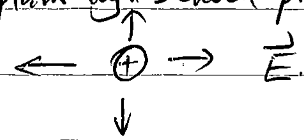
Divide into N bundle, each bundle

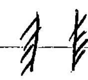
$$W = \frac{dQ_{e1}}{N}$$

Determine how many bundle is absorbed by A_2.

force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Explain high school physics





$E = V/d$

EM Waves

$\vec{E}$ — electric field V/m

$\vec{H}$ — magnetic field A/m $(= \frac{C}{s \cdot m})$ due to current flow

When an atom is under an electric field

$\longrightarrow E$

⊕ ⊖ — polarization

dipole $C \cdot m$

Polarization (per unit volume)

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

$\left[\frac{C \cdot m}{m^3} = \frac{C}{m^2}\right]$ └ susceptibility

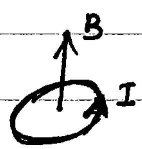
Vaccum susceptibility $8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

displacement

$$\vec{D} = \vec{P} + \varepsilon_0 \vec{E} = \varepsilon_0 (1+\chi) \vec{E} = \varepsilon \vec{E}$$

magnetic induction $\vec{B} = \mu \vec{H}$

↑ permitivity.



vacuum $\mu_0 = 4\pi \times 10^{-7} \frac{NS^2}{C^2} = \frac{H}{m}$

$\vec{B}$ & $\vec{E}$ pair physically

$\vec{D}$ & $\vec{H}$ pair

In reality, people used to treat $\vec{E}$ & $\vec{H}$, $\vec{B}$ & $\vec{D}$

$$\varepsilon_1(T_1) = \frac{\int_0^\infty \int_\Omega \varepsilon'_{\lambda,1}(\lambda,\theta_1,T_1)\, i'_{\lambda b}(\lambda,T_1)\cos\theta\, d\Omega\, d\lambda}{\sigma T_1^4}$$

We divide $dQ_{e,1}$ into $\underline{N}$ bundles, the energy of each bundle

$$W = \frac{dQ_{e,1}}{N}$$

Follow each bundle, if S bundle is absorbed at A_2, then

$$dQ_{12} = W S_2 \qquad (\text{since } T_2 = 0)$$

$$= \frac{\varepsilon_1(T_1)\sigma T_1^4 dA_1}{N} S_2 .$$

(2) How to determine directi & wavelegth of each bundle

$\varepsilon'_{\lambda,1}$, T_1, obey properties & law

in directi θ_1, $d\theta_1$, $\varphi = 2\pi$, power

$$dP'_\lambda = \underset{\uparrow}{2\pi}\, i'_{\lambda b}(\lambda,T_1)\cdot \varepsilon'_\lambda \cdot \cos\theta_1 dA_1\, \underline{\sin\theta_1 d\theta_1}\, d\lambda$$

comes from φ_1 independent.

probability $\quad p(\lambda,\theta_1)\,d\theta\,d\lambda = \dfrac{2\pi\, \varepsilon'_\lambda\, i'_{\lambda b} \cos\theta_1 \sin\theta_1\, d\theta_1\, d\lambda}{\varepsilon_1 \sigma T_1^4}$

Random #.
Identical chance

$p(\lambda,\theta_1)$ — probability density (distributi function) $[0,1]$

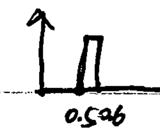
~~cumulative distributi funct~~

$$\Rightarrow \quad p(\lambda)\,d\lambda = \int_0^{\frac{\pi}{2}} p(\lambda,\theta_1)\,d\theta_1$$

similarly $p(\theta)$.

give a random number $\quad p(\lambda) = R_\lambda \Rightarrow \lambda.$

or $\quad R_\theta \Rightarrow \theta.$



r_λ may not have any match with λ.

Another way — cummulative distributi functi

$$\int_{-\infty}^{\lambda} P_\lambda(\lambda)\, d\lambda \qquad [0,1]$$

specify φ_1 directi $\qquad \varphi_1 = 2\pi R_{\varphi_1}$

└ random $[0,1]$

given $\qquad R_\lambda, R_{\theta_1}, R_{\varphi_1} \Rightarrow \lambda, \theta_1, \varphi_1 \Rightarrow$ strikes A_2 or not.

whether absorbed or not $\qquad \varepsilon_2'(\lambda,\theta_2) = \alpha_2'(\lambda,\theta_2)$

another random number $\qquad R_{a2}$

If this true $R_{a2} \le \alpha_2'(\lambda,\theta_2)$

bundle is absorbed.

Maxwell equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{Faraday's law}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \qquad \left(\text{if no } \frac{\partial \vec{D}}{\partial t} \text{ term. Ampere's law}\right)$$

↑ Maxwell's contributi displacement current

$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial k}$

∇^2 — Laplace operator

$$\nabla \cdot \vec{D} = \rho_{net} \qquad \text{Coulomb's law}$$

$$\nabla \cdot \vec{B} = 0 \qquad \text{(no free magnetic poles)}$$

$$\vec{J} = \sigma \vec{E} \qquad \text{Ohm's law}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E}\right)$$

⇓

$$\nabla(\nabla \cdot \vec{E}) - (\nabla \cdot \nabla)\vec{E}$$

⇓ if $\rho_{net} = 0$

$$\nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t}$$

$$\sigma = 0 \Rightarrow \quad \nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2}$$

wave equation.

$$\vec{E} = \vec{E}_0 \cos(\omega t - k \cdot x) \quad \text{— propagati along } x$$

$$-\vec{E}_0 k^2 \cos(\omega t - kx) = \varepsilon\mu \vec{E}_0 \omega^2 \cos(\omega t - kx)$$

$$\frac{\omega}{k} = \pm \frac{1}{\sqrt{\varepsilon\mu}}$$

$$\omega = 2\pi\nu, \qquad k = \frac{2\pi}{\lambda}.$$

$$\nu \cdot \lambda = \frac{1}{\sqrt{8.85\times10^{-12} \times 4\pi\times10^{-7}}} = 3\times10^{8}\ \text{m/s} =$$

If we assumed $\vec{E} = \vec{E}_0 \sin(\omega t - kx)$ same results

Easier to deal with

$$\vec{E}_c = E_0\, e^{-i(\omega t - kx)}$$

If not propagating along x

$$\vec{E}_c = \vec{E}_0\, e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

↑ field direct. ↑ wave vector.

$$\vec{E} \perp \vec{H} \perp \vec{k}$$

∴ right hand rule. (Comment to ~~neg~~ left hand materi

Consider $\sigma \neq 0$

$$\vec{k}\cdot\vec{k} = \mu\omega^2 [\varepsilon_0(1+\chi) + i\sigma/\omega] = \mu\tilde{\varepsilon}\,\omega^2$$

↑ complex permit

$$|k|^2 = \frac{\omega^2}{\cancel{c^2}\mu\tilde{\varepsilon}} \qquad c = \frac{1}{\sqrt{\mu\tilde{\varepsilon}}}$$

Complex Refractive index $N = \frac{c_0}{c} = \sqrt{\frac{\tilde{\varepsilon}\mu}{\varepsilon_0\mu_0}}$