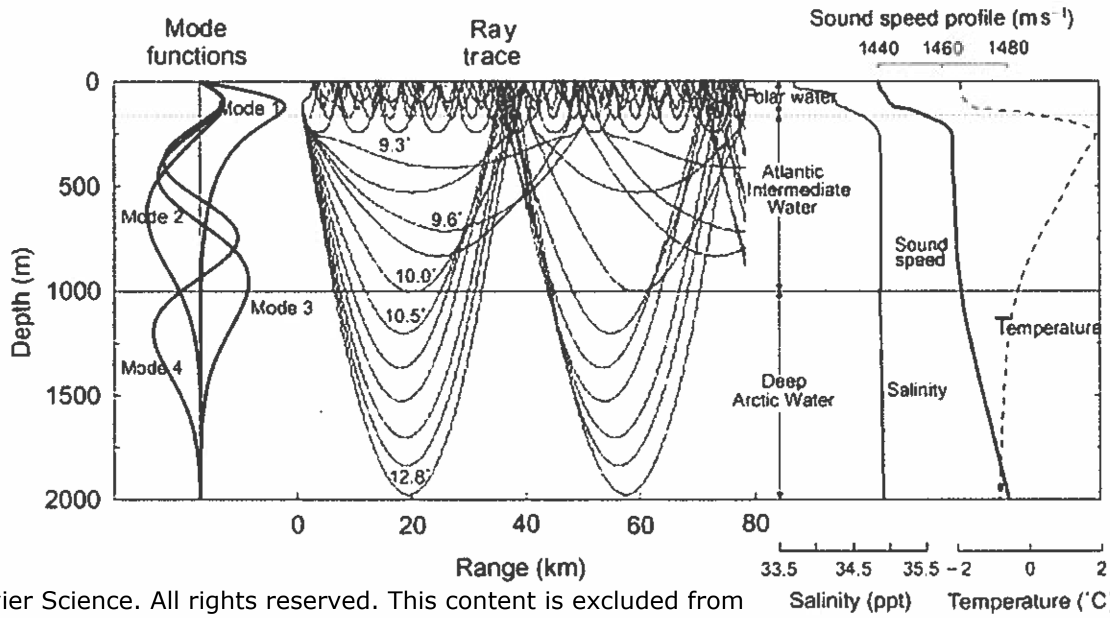
HWK

Problem Set #2. (Due in two weeks from assignment date)



The above figure, from Mikhalevsky's article in the Encyclopedia of Ocean Sciences, show the rays and modes produced (at a given frequency for the modes) by an Arctic profile with a fresh water surface layer. This "halocline layer", which causes a strong acoustic duct, is very important to climate science. Specifically, this layer seems to be thinning, which allows the warmer water just below it (the Atlantic Intermediate Water) to access the ice on the surface and melt it. And, as all polar bears and most climate scientists would agree, melting the earth's polar ice caps is maybe not such a good thing. (Note that this is an oceanic warming effect, caused by less fresh water runoff by rivers, and is only indirectly tied to the atmosphere through rainfall levels.)

One method that has been proposed to measure this surface fresh water layer is "acoustic halinometry", in which we measure salinity differences from a background profile. The material we just covered in the ray theory lecture is absolutely perfect for looking at this problem, and conversely, this problem is well adapted to looking at aspects of ray theory. Win-win.

1) Jumping to the answer if you have the c(z) profile.

Let's further simplify the profiles above, in the spirit of "back of the envelope inverse theory". Let's assume the water temperature is T=0 deg C everywhere to begin with. Also, let's assume that the salinity is 35 ppt everywhere. Then the "background profile" for the soundspeed would just be $c(z)=1449.2 + 0.0165z$ where $c(z)$ is in m/s and z is in m. Now, let's add the soundspeed perturbation due to the halocline layer. We see that we have a (roughly) linear gradient layer going from 1440 m/s at the surface to ~1460 m/s at 250 m depth. From the equation of state, see what this predicts the salinity perturbation should be, and then compare it to the real answer in the graph. They should jive reasonably well. This is a trivial inverse, but it works if someone has handed you the soundspeed profile a priori.

2) Now we can turn to the more realistic inverse problem using acoustics, where nobody hands us the perturbed profile. But we can start with a "savvy guess" about the background profile, which we again take as $c(z)=1449.2 + 0.0165z$ where $c(z)$ is in m/s and z is in m. Now consider a 250 Hz, 100 Hz bandwidth acoustic source at $z=1$ m (essentially at the surface) and a receiver 100 km away, also at $z=1$ m . The water depth can be taken as 4000 m (deep Arctic).

    a) Using the "ray equations" we saw in class, integrate them directly to get a feel for the (circular arc!) propagation paths. Do a few (non bottom interacting) angles to get a feel for how things work. Use the simplest integration scheme you can! (No eigenrays for this part!)
    b) Let's try another method to get rays. Using the fact that for constant gradient conditions, the rays are arcs of circles, find the non-bottom interacting eigenrays for the system. (Surface interaction is fair game!)
    c) Now, let's use a modern computational tool. Use BELLHOP to get what the non-bottom interacting rays are, and also their travel times and intensities. Make a stick plot of the travel times/intensities. (That is, time on the x-axis and intensity on the y-axis, with each n-th arrival being shown as a vertical "stick" of intensity I_n at travel time t_n.
3) Now, the real profile comes into play. In a real-world experiment, we would measure the real ray travel times, see what their differences were with our predictions from the last section, and then use that "difference data" in an inverse scheme (to be treated later in the course) to get the real c(z) profile. For now, let's just generate "artificial data" by taking the double-ducted c(z) profile from problem #1 and running BELLHOP on it to get eigenrays for the geometry described in problem #2. What does the eigenray structure look like now? Describe the qualitative differences between the real and background cases. It is apparent that our simple "difference between raypaths" scheme is going to be difficult to implement at low grazing angles. Any guess as to what's happening in this regime and how we might save our "differencing" scheme?
4) We said that our source in the first three problems was 250 Hz, with 100 Hz bandwidth (BW). We can resolve rays in time when their arrival time difference is $\Delta t \geq 1/BW$. Which rays are resolved for the background and the real profile?
5) The "vertical resolution" that each ray can achieve is given (roughly) by the difference between the surface and lower ray turning points, for our particular example. What are these bottom turning points (from the BELLHOP output)? When combined, the vertical resolution is the distance between neighboring bottom turning points for the resolved rays. What are these? Is this good resolution for oceanography?!?

Extra credit (One point each)

1) What is the Fresnel zone of the rays, if we just assume a straight line between source and receiver in the above?
2) What is the Fresnel zone width of the rays if we use the wavefront curvature equation I showed in class? (Hint: just involves a second derivative of the circular arc.)
3) Can a Gaussian Beam mimic the Fresnel Zone's behavior? (Incentive to read that section!)

2.682 Acoustical Oceanography
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