Lecture #12: Background

Underwater sound propagation occurs in a medium with inherently rough boundaries, as any seagoing oceanographer can attest. Yet all too often students work on projects which have strong scattering components, but have never seen scattering theory in a formal course. And while one lecture is obviously inadequate to cover such a vast area, it is certainly good enough to give a student a feel for what the area looks like in general, as well as a few simple tools.

There is a list of topics in the instructor's notes, but due to space/time limitations, the last three starred topics were not covered in this course. Two references, Ogilvie and B&L are cited here, which cover much of the material (though not all). An older book by Tolstoy and Clay is also used.

The first topic, the Rayleigh parameter for roughness, is probably the most physical definition of roughness one can find. This parameter gets used extensively in the theory. Note, however, that it deals with the rough surface's height, but not its slope. There is a horizontal scale missing!

Huygens Principle, that each point on the rough surface acts as a radiator of scattered waves is another "Physics 101" concept, and can be implemented easily on a computer to get some idea of scattering from a surface.

The section on the "Statistics of Rough Surfaces" is elementary, but useful. The mean, variance, Gaussian statistics, and the Central Limit Theorem should all be familiar to students at this level.

The surface correlation function for a rough surface is the next piece we need to characterize roughness – it is the horizontal scale of the roughness (that the Rayleigh parameter ignored). Various useful parameterization of the surface roughness correlation are discussed here.

The" characteristic function" (Fourier transform of the surface height distribution) is shown next, and has some utility, as we'll see. Of far more utility in scattering theory is the power spectrum of the surface, the Fourier transform of the unnormalized correlation function. This descriptor has both the surface height description and the correlation scales folded into it, as shown. The integral over the wave spectrum gives the variance of the surface height. Examples of power spectra for anisotropic roughness and gaussian and exponential distributions are shown.

Means and variances are the most basic moments/statistics to consider, but as one digs deeper, there are higher moments and other descriptions to consider. Two point height probability distributions, surface derivatives, and the radius of curvature are all discussed briefly.

Three very important concepts for all random medium studies (not just our ocean surfaces) are isotropy, stationarity, and ergodicity. These are explained decently by the notes, with examples. As a "real life" note, I am working currently on some acoustic intensity fluctuation statistics due to combinations of ocean and seabed process "randomness." One interesting piece of this work is that the "stationary time" one needs to consider when making statistical averages is a mixture of the times characteristic of the processes and their relative strengths, and is not something one can "guesstimate" trivially.

Fractal surfaces are discussed next, and of note here is the "Goff-Jordan" spectrum for bottom roughness, devised by John Goff and Tom Jordan of MIT. (John is now a geologist at U. Texas, my old haunt, and frequently collaborates with acousticians working on how sound interacts with the seabed.).

A small section on probability integrals concludes the "basic review" section. These (Gaussian) moment integrals are simple to evaluate and useful.

We now come to where we can do some real scattering theory. Following B&L, we first look at the "method of small perturbations" (MSP), in which the boundary conditions at the rough surface $\zeta(\vec{r})$ are transferred to the mean surface by expending them in a power series in small parameter λ (thus the "small"). This is done for both pressure release and rigid bottom B.C.'s. Going through the math, we obtain simple form (s) for the scattered pressure p_{scatt}, Eqs. 9.2.12 and 9.2.13 of B&L. If we then express $\zeta(\vec{r})$ as a Fourier integral over wave numbers, we get a very neat form for p_{scatt}, where the rough surface gives rise to scattered plane waves that satisfy the Bragg law scattering condition. (This condition pops up with frightening regularity in scattering theory!)

Though $\langle\, p_{scatt}\rangle = 0$ for MSP, the scattered intensity, (the second moment of p_{scatt}) should not be zero. A simple "square and average" calculation using the previous results gives a surprisingly simple equation, $I_{scatt} = 4\gamma_0^2\sigma^2$.

The next piece we treat is the near field versus far field, and a simple criterion is developed for the transition distance. Basic stuff, but useful, as we generally look at far field scattering approximations first - the near field is messy.

The notes, "Back to the Far Zone" are from B&L, and look at scattering from a patch of rough surface in the far field using our MSP results. Of note in this section are: 1) the definition of the surface scattering coefficient m_s, (eq. 9.3.7 B&L is worth remembering), 2) its evaluation in the MSP, 3) the fact that the scattering picks out the Bragg resonant components, 4) the simple expression for backscatter (needed in ocean acoustics for studying reverberation from active

sources!), and 5) the curious fact that specular reflection just depends on the mean surface! The expressions derived also provide a way to measure the power spectrum of the rough surface.

Continuing in B&L section 9.7, we next discuss the so called "Kirchoff approximation" or "tangent plane approximation," which is good for large roughness, but small slopes.   (The complement of MSP! All we're lacking is large roughness and large slopes, which is much more difficult!). To calculate the scattered field, this work starts from the Helmholtz Integral Formula (HIF) which is an exact solution! (The derivation for the HIF can be found in Medwin and Clay and other ocean acoustics books. It is a standard vector calculus result.) The first order of business is to explicitly evaluate the various terms in the HIF. We can then average over realizations. For an infinite surface and plane waves, one gets the old result $\langle p_s \rangle = V_c p_0$ where V_c is obtained from a simple Fourier transform of the surface height spectrum. The case for a finite patch of rough surface scattering plane waves and an observer in the farfield is treated next, and again one expends the effort evaluating individual terms, as well as making the farfield (simplifying) approximation.

So far, our discussion of rough surface scattering has all been for "free space." But we're interested in ocean wave guides, yes? So, back to rays and modes! In the ray picture, we can pretty much use the theory derived before for each local surface interaction ("bounce", in the jargon), so no real need to belabor things. The mode picture is different, and we will appeal to range dependent mode theory (adiabatic and coupled) to describe the roughness effects.

Our first look at the rough modal waveguide is from Tolstoy and Clay, an older text book, but one with many nice "basics" in it. The beginning of section 6.9 of T&C is old stuff, up through Eq. 6.98. In Eq. 6.100, the breakup of the $k_m(r)$ into an (average) background wave number and a range varying perturbation, $\varepsilon_m(r)$, is where the phase part of the roughness scattering enters explicitly. From this, one easily gets the roughness induced phase accumulation with range (the integral of $\varepsilon_m(r)$). This latter quantity gives the phase fluctuation one sees, mode by mode, at the receiver. The amplitude fluctuation from the roughness comes primarily from the changes in the mode function at the source and receiver, Eq. 6.99. Combining 6.99 and 6.100, we get the adiabatic "rough surface" pressure field in mode theory, Eq. 6.104. So, are we there yet? Of course not!

The surface and bottom roughness are generally treated as random media (via power spectra), whereas our result above is deterministic, i.e., it uses a specific realization of the roughness. We thus need to get the distribution of the surface roughness (gaussian for surface waves, and we can fake gaussianity for the bottom roughness for now), and relate it to the $\varepsilon_m(r)$. Simple estimates show that gaussian roughness produces "pretty durn close to" gaussian

$\varepsilon_m(r)$ (try this for the hard bottom!), so we can buy off on T&C's Eqs. 6.105 a, b. We next assume spatial stationarity of the roughness to allow us to evaluate the correlation function easily. Just the relative coordinate, $(x - x')$, enters here, not the absolute position coordinates. This gives Eq. 6.107. As always, when in doubt, "buy a Gaussian", and so a gaussian surface correlation function is invoked in Eq. 6.108. This gives an extremely simple result in Eq. 6.109, that the variance of the (range integral) phase fluctuation is proportional to the range, the correlation length, and the variance of the eigenvalues. Equations 6.110 to 6.112 show how the phase fluctuations decrease the coherence of the pressure field with range, due to the "attenuation" factor (ala Eq. 6.109) introduced. The rest of the material from T&C past Eq. 6.112 is about higher order correlations, and gets a bit more involved, so I will let the reader browse and decide if it is worth looking at in detail.

To conclude, there is a huge literature on rough surface scattering (e.g. the radar scattering literature just for starters), and again I must apologize for this very brief treatment of a vast and fascinating subject.

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