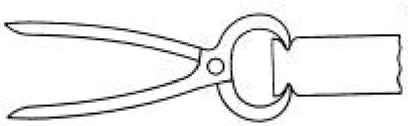
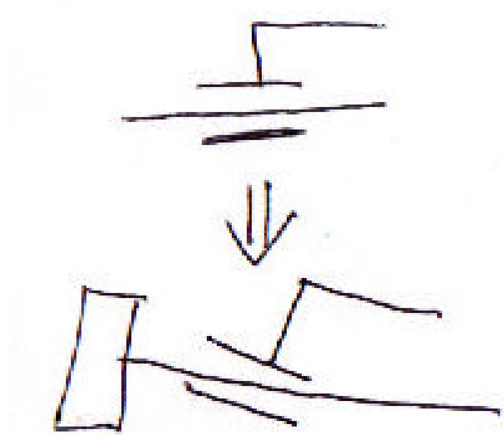
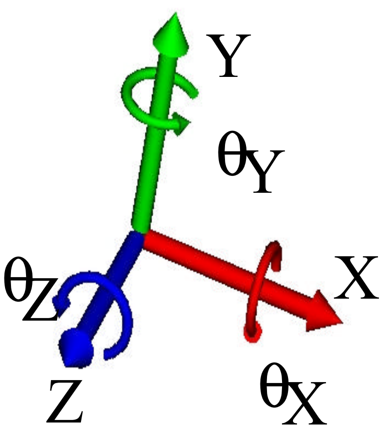


Topic 3
Fundamental Principles

Topics

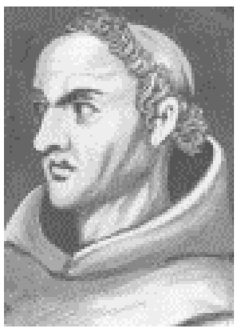
- Occam's Razor: KISS & MISS
- Saint-Venant's Principle
- Abbe's Principle
- Maxwell's Reciprocity
- Self-Principles
- Stability
- Symmetry
- Superposition
- Golden Rectangle
- Parallel Axis Theorem
- Accuracy, Repeatability, Resolution
- Sensitive Directions
- Reference Features
- Structural Loop
- Free Body Diagram
- Centers of Action
- Exact Constraint Design
- Elastic Averaging
- Dimensional Analysis
- Stick Figures



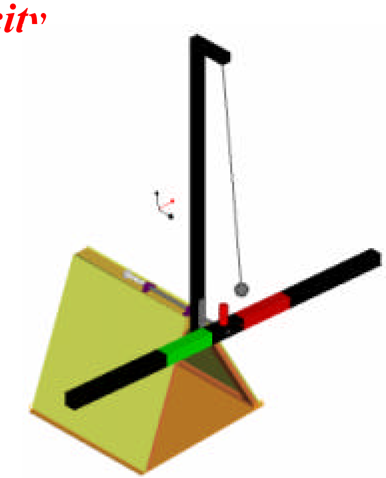


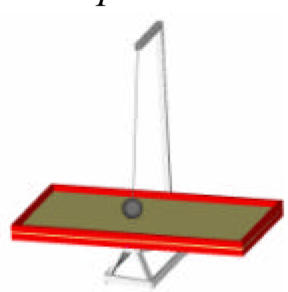
Occam's Razor: KISS & MISS

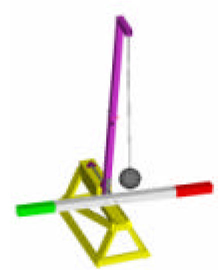
- William of Occam (or Ockham) (1284-1347) was an English philosopher and theologian
  - Ockham stressed the Aristotelian principle that *entities must not be multiplied beyond what is necessary*
  - "Ockham wrote fervently against the papacy in a series of treatises on papal power and civil sovereignty. The medieval rule of parsimony, or principle of economy, frequently used by Ockham came to be known as **Ockham's razor**. The rule, which said that *plurality should not be assumed without necessity* (or, in modern English, *keep it simple, stupid*), was used to eliminate many pseudo-explanatory entities" (http://wotug.ukc.ac.uk/parallel/www/occam/occam-bio.html)
    - ***A problem should be stated in its basic and simplest terms***
    - ***The simplest theory that fits the facts of a problem is the one that should be selected***
    - ***Limit Analysis is an invaluable way to identify and check simplicity***



- Use fundamental principles as catalysts to help you
  - Keep It Super Simple
  - Make It Super Simple
  - Because *"Silicon is cheaper than cast iron…"*(Don Blomquist)

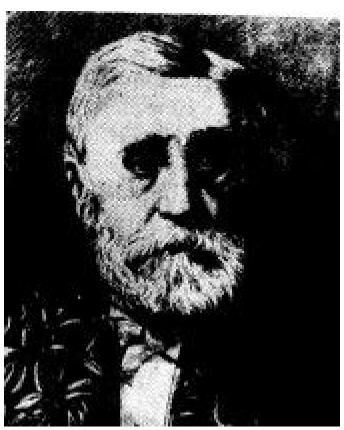




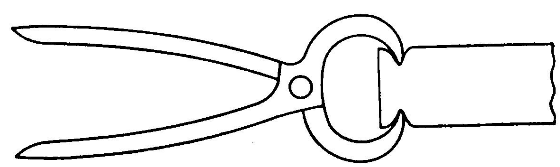


Saint-Venant's Principle

- Saint-Venant's Principle
  - Saint-Venant did extensive research in the theory of elasticity, and many times he relied on the assumption that local effects of loading do not affect global strains
    - e.g., bending strains at the root of a cantilever are not influenced by the local deformations of a point load applied to the end of a cantilever
  - The engineering application of his general observations are profound for the development of conceptual ideas and initial layouts of designs:
    - To NOT be affected by local deformations of a force, be several characteristic dimensions away
      - On the city bus, how many seats away from the smelly old drunk do you want to be?
    - To have control of an object, apply constraints over several characteristic dimensions
      - These are just initial layout guidelines, and designs must be optimized using closed-form or finite element analysis

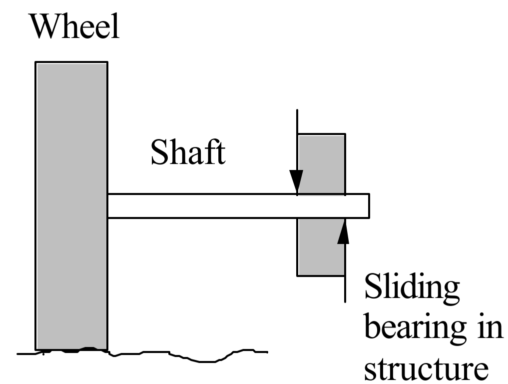


Barré de Saint-Venant 1797-1886

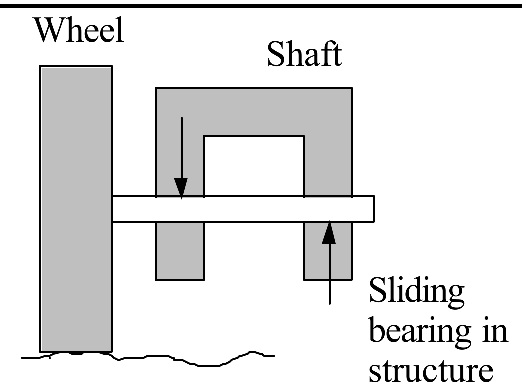


Saint-Venant's Principle: *Bearings*

- Saint-Venant: *Linear Bearings:*
  - L/D>1, 1.6:1 very good, 3:1 super ideal
  - Every year some students try to make L/D<1 and their machines jam!
    - Wide drawers guided at the outside edges can jam
    - Wide drawers guided by a central runner do not!
- Saint-Venant: *Rotary Bearings:*
  - L/D>3 if you are to have the bearings "build the shaft into a wall"
  - IF L/D<3, BE careful that slope from shaft bending does not KILL the bearing!
- Bolting bearings in pace: beware the zone under a bolt that deforms due to bolt pressure!



!!Non Optimal!!



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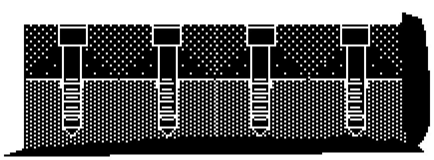
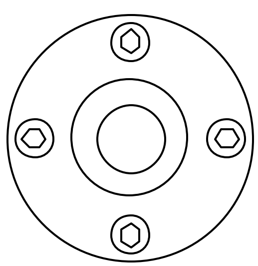
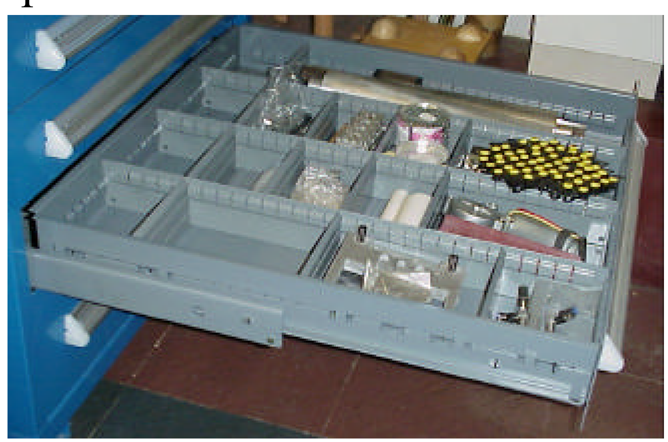
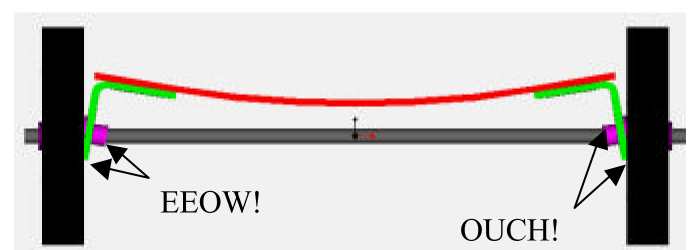


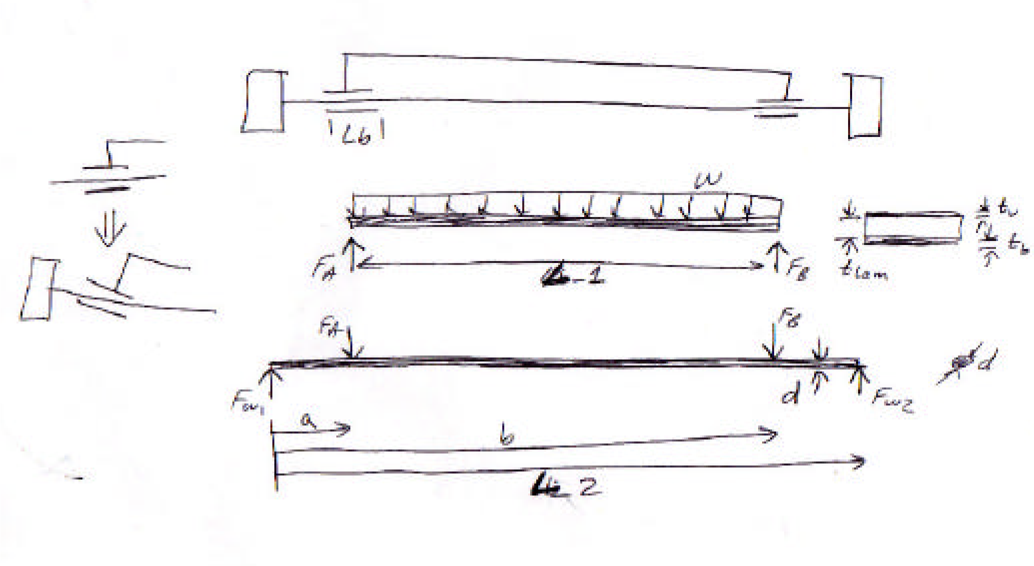


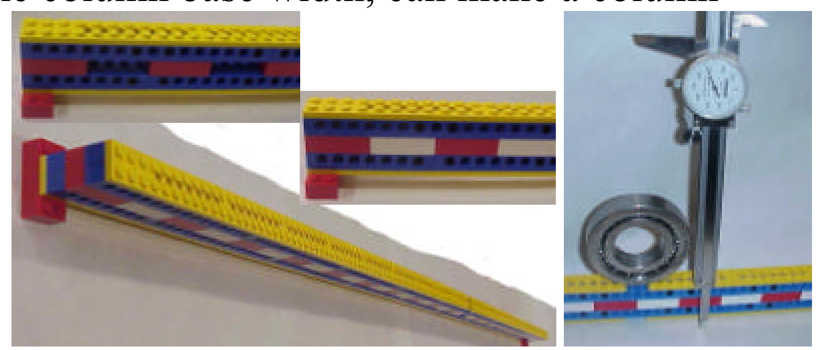


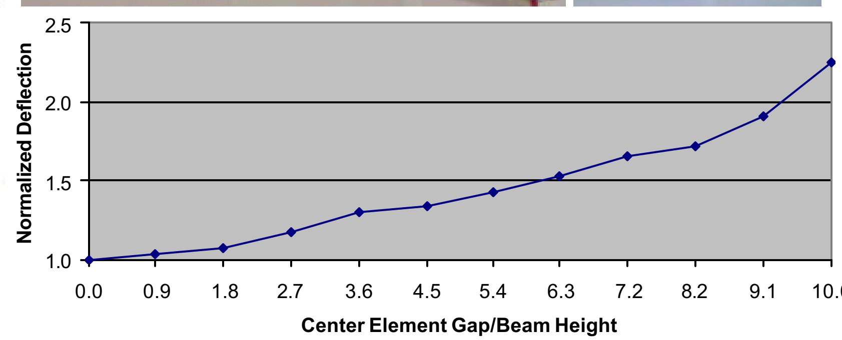


Saint-Venant's Principle: *Structures*

- To NOT feel something's effects, be several characteristic dimensions away!
  - If a plate is 5 mm thick and a bolt passes through it, you should be 3 plate thicknesses away from the bolt force to not cause any warping of the plate!
    - Many bearing systems fail because bolts are too close to the bearings
- To DOMINATE and CONTROL something, control several characteristic dimensions
  - If a column is to be cantilevered, the anchor region should be 3 times the column base area
  - Most machines that suffer from "lawn furniture syndrome" have inadequate anchoring
    - Diagonal braces or gussets, that are 3-5 x the column base width, can make a column appear to be cantilevered

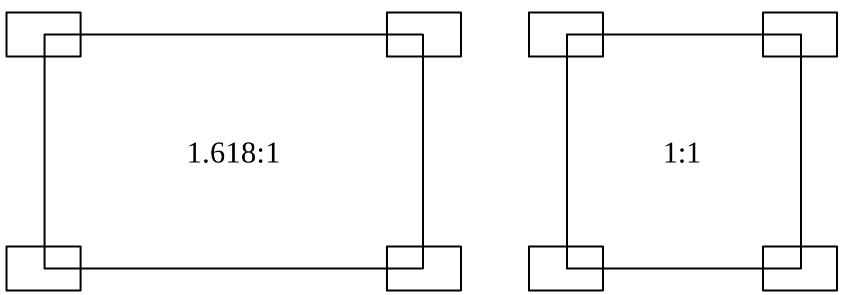






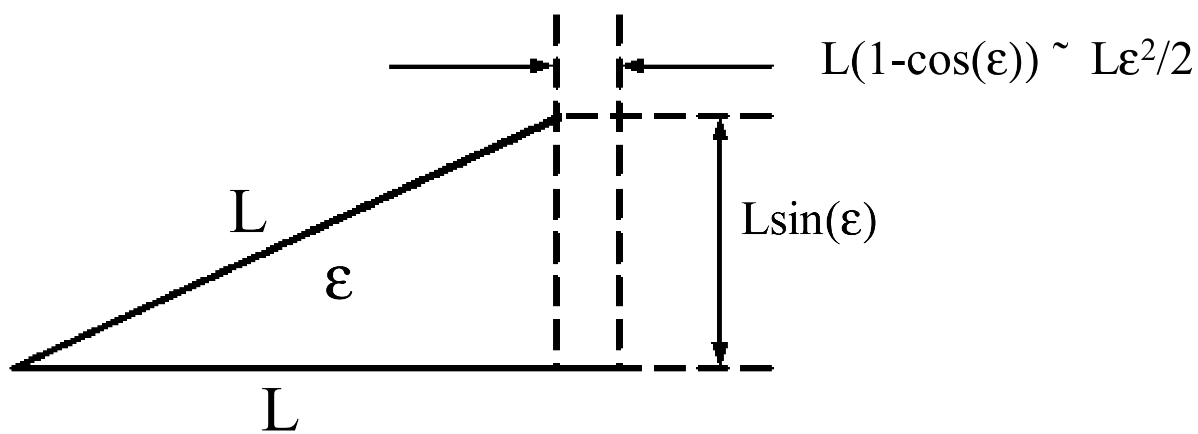
The Golden Rectangle

- The proportions of the *Golden Rectangle* are a natural starting point for preliminary sizing of structures and elements
  - *Golden Rectangle:* A rectangle where when a square is cut from the rectangle, the remaining rectangle has the same proportions as the original rectangle
  - Watch *Donald in Mathmagic Land!*
- Example: Bearings:
  - The greater the ratio of the longitudinal to latitudinal (length to width) spacing:
    - The smoother the motion will be and the less the chance of walking (yaw error)
    - First try to design the system so the ratio of the longitudinal to latitudinal spacing of bearing elements is about 2:1
    - For the space conscious, the bearing elements can lie on the perimeter of a golden rectangle (ratio about 1.618:1)
    - The minimum length to width ratio is 1:1 to minimize yaw error
    - The higher the speed, the higher the length to width ratio should be



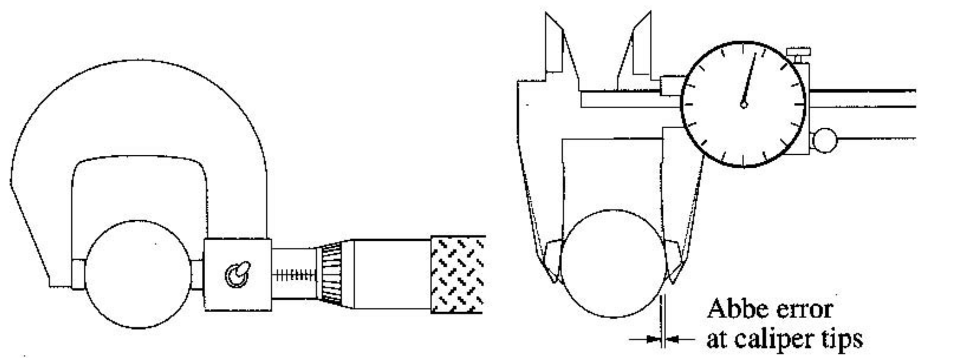
Abbe's Principle

- In the late 1800s, Dr. Ernst Abbe (1840-1905) and Dr. Carl Zeiss (1816-1888) worked together to create one of the world's foremost precision optics companies
- The Abbe Principle (Abbe errors) resulted from observations about measurement errors:
  - *If errors in parallax are to be avoided, the measuring system must be placed coaxially with the axis along which the displacement is to be measured on the workpiece*
    - *Strictly speaking, the term Abbe error only applies to measurement errors*
- When an angular error is amplified by a distance, to create an error in a machine's position, for example, the strict definition of the error is a sine or cosine error



Abbe's Principle: *Locating Components*

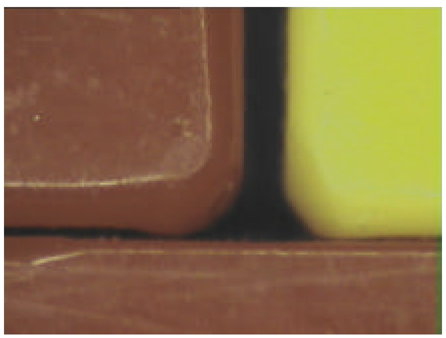
- Geometric: Angular errors are amplified by the distance from the source
  - Measure near the source, and move the bearings and actuator near the work!
- Thermal: Temperatures are harder to measure further from the source
  - Measure near the source!

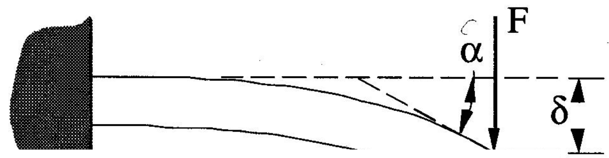


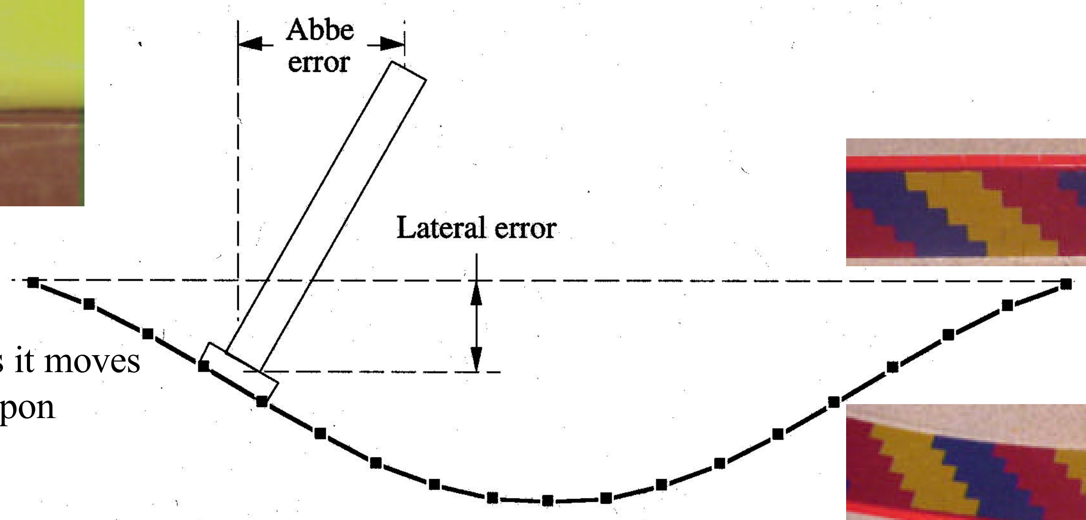
- Thinking of Abbe errors, and the system FRs is a powerful catalyst to help develop DPs, where location of motion axes is depicted schematically
  - Example: Stick figures with arrows indicating motions are a powerful simple

Abbe's Principle: *Cascading Errors*

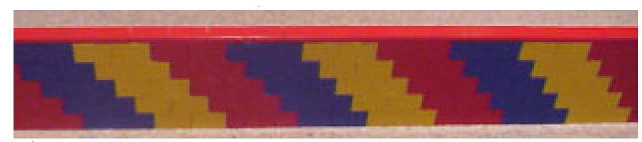
- A small angular deflection in one part of a machine quickly grows as subsequent layers of machine are stacked upon it…
  - A component that tips on top of a component that tips…
  - If you give a mouse a cookie…..
- Designs must consider not only linear deflections, but angular deflections and their resulting Abbe Errors…





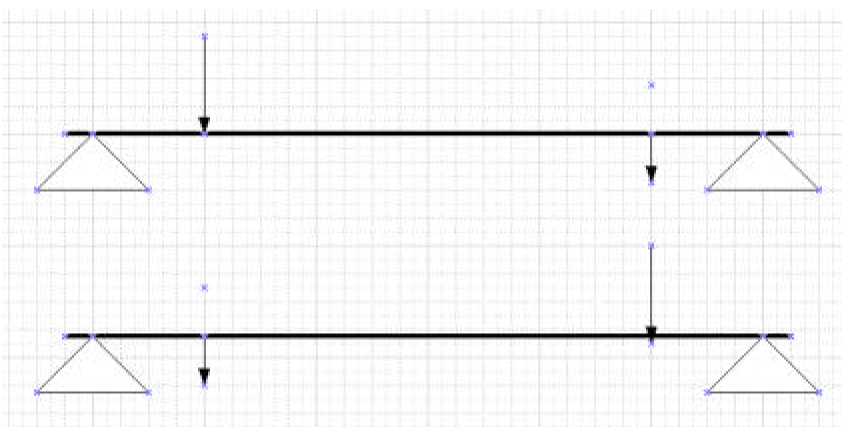


Motion of a column as it moves and deflects the axis upon which it rides



Maxwell's Reciprocity

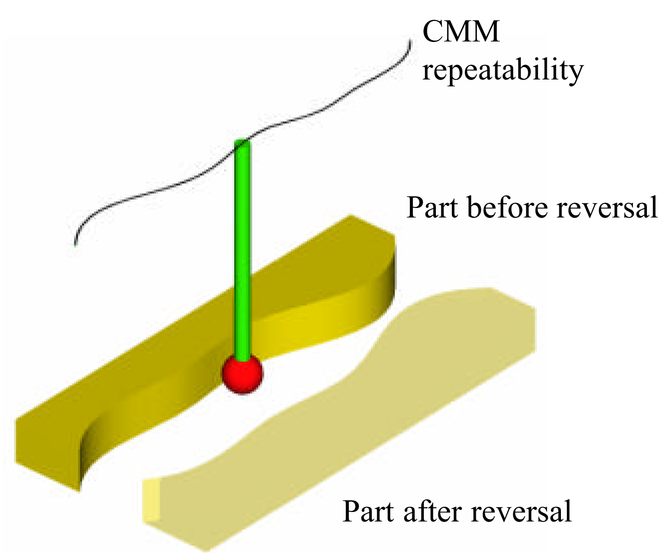
- Maxwell's theory of *Reciprocity*
  - Let *A* and *B* be any two points of an elastic system. Let the displacement of *B* in any direction *U* due to a force *P* acting in any direction *V* at *A* be *u*; and the displacement of *A* in the direction *V* due to a force *Q* acting in the direction *U* at *B* be *v*. Then $Pv = Qu$ (from Roark and Young Formulas for Stress and Strain)
- The principle of *reciprocity* can be extended in philosophical terms to have a profound effect on measurement and development of concepts
  - Reversal
  - Critical Thinking



James Clerk Maxwell 1831-1879

Reciprocity: *Reversal*

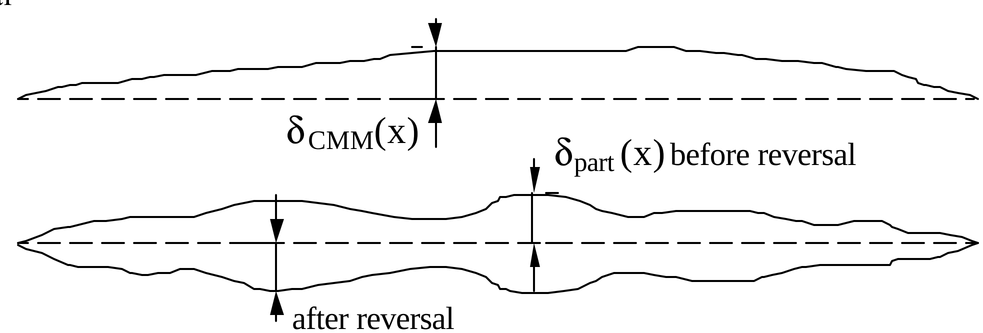
- A method that is used to take out repeatable measuring instrument errors from the measurement
  - See ANSI standards for axis of rotation, straightness and machine tool metrology for excellent tutorials on applying reciprocity to measurement!
- One of the principal methods by which advances in accuracy of mechanical components have been continually made
- There are many application variations for measurement and manufacturing
  - Two bearings rails ground side-by-side can be installed end-to-end
  - A carriage whose bearings are spaced one rail segment apart will not pitch or roll



$$Z_{\text{probe before reversal}}(x) = \delta_{CMM}(x) - \delta_{part}(x)$$

$$Z_{\text{probe after reversal}}(x) = \delta_{CMM}(x) + \delta_{part}(x)$$

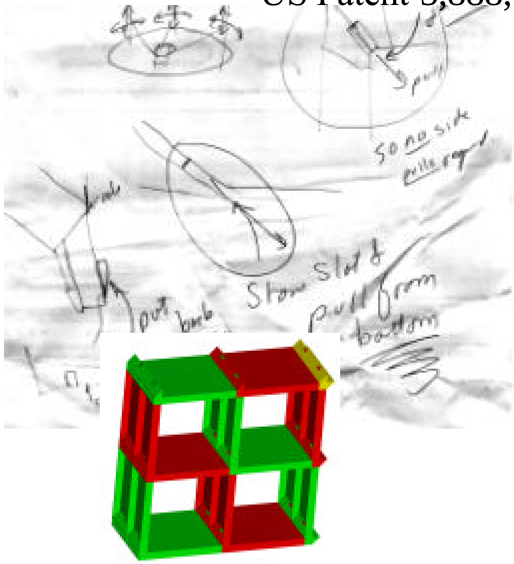
$$\delta_{part}(x) = \frac{-Z_{\text{probe before reversal}}(x) + Z_{\text{probe after reversal}}(x)}{2}$$

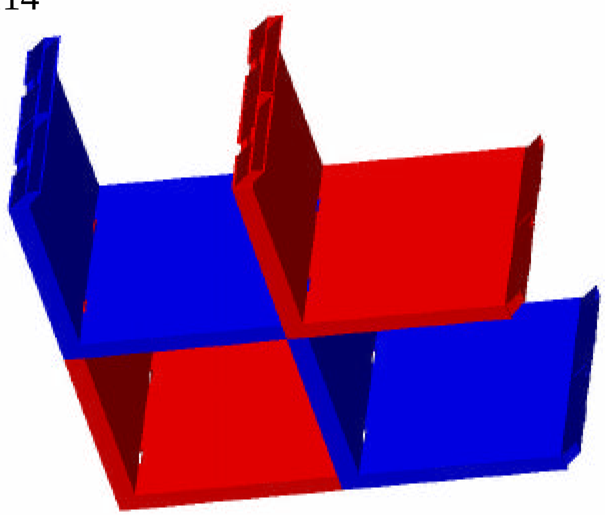


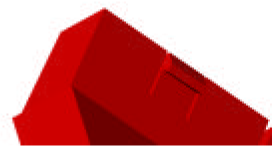
Reciprocity: *Critical Thinking*

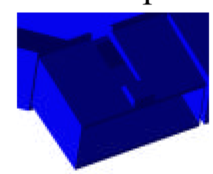
- If you are happy with something, turn it around!
- If you are unhappy with something, turn it around!
- If you are comfortable on your back, you should still turn over and try lying down on your front……
- You can make a system *insensitive* to its surroundings, or you can *isolate* a system from its surroundings…
- If you cannot solve a problem by starting at the beginning, start at the end and work backwards!
- Example: Snap fit that required a side pull in a mold was literally inversed to eliminate the need for a side pull!
  - CubbeeZ™ modular storage elements
  - US Patent 5,888,114

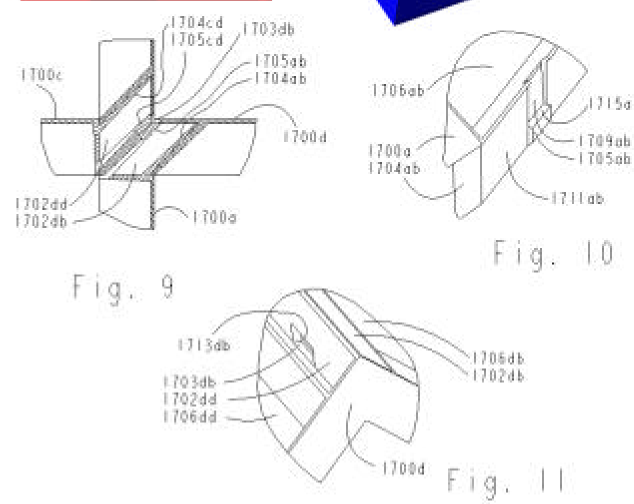
Side pull => No side pull





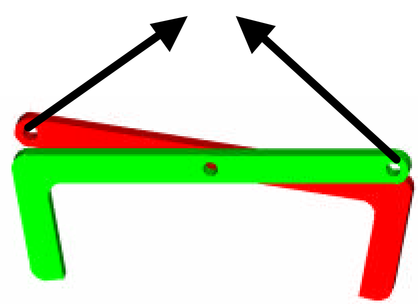






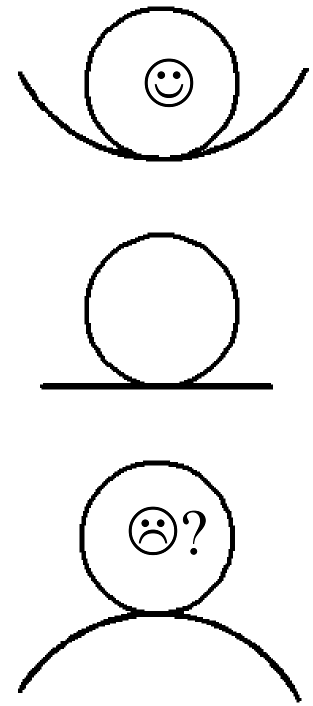
Self Principles

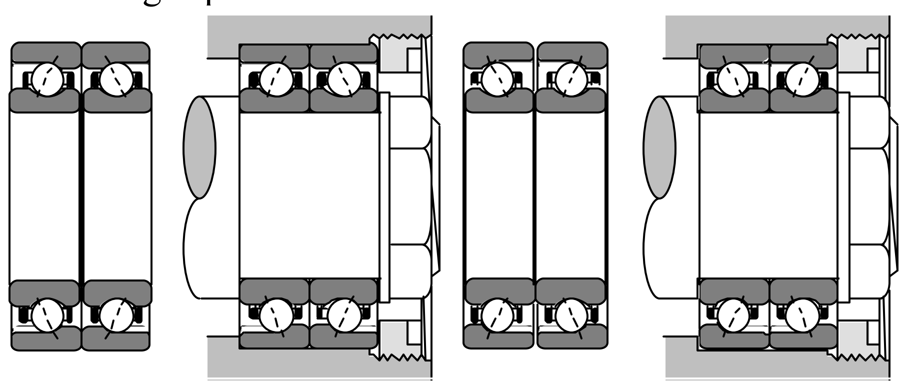
- The manner in which a design reacts to inputs determines its output
  - How a design thus affects itself, can be used to advantage
  - Reciprocity would philosophically tell us to look for a solution where a potentially detrimental result can be used to cancel the effect
  - Martial artists practice this principle all the time!
- *Self-Help:* A design that uses the inputs to assist in achieving the desired output
  - An initial effect is used to make the device ready for inputs
  - The supplemantary effect is that which is induced by the inputs, and it enhances the output
- Example: Airliner doors act like tapered plugs
  - When the door is shut, latches squeeze the seal, making the cabin airtight
  - As the plane ascends and the outside air pressure decreases, the higher inner air pressure presses on the door and causes the seal to seal even tighter
  - Back-to-back angular contact bearings are thermally stable
- Other self principles similarly exist:
  - Self Balancing, Self-Reinforcing, Self-Protecting, Self-Limiting, Self-Damaging, Self-Braking, Self-Starting….



Stability

- All systems are either stable, neutral, or unstable
  – Saint-Venant's principle was applied to bearing design to reduce the chance of sliding instability (e.g., a drawer jamming)
  – A snap-fit uses an applied force to move from a stable, to a neutrally stable, to an unstable to a final new stable position
  – Wheels allow a system to roll along a flat surface
  – As the load on a tall column increases, infintesimal lateral deflections are acted on by the axial force to become bending moments, which increase the deflections….
    - Reciprocity says this detrimental effect can be useful: fire sprinklers are activated by a column that buckles when it becomes soft…
  – Bearings mounted in the *back-to-back* mode use axial thermal growth to cancel radial thermal growth and thus remain thermally stable at high speeds
  – Bearings mounted in the *face-to-face* mode are more tolerant of misalignments, but axial thermal growth effectively adds to radial thermal growth and causes the bearings to overload and seize at high speeds

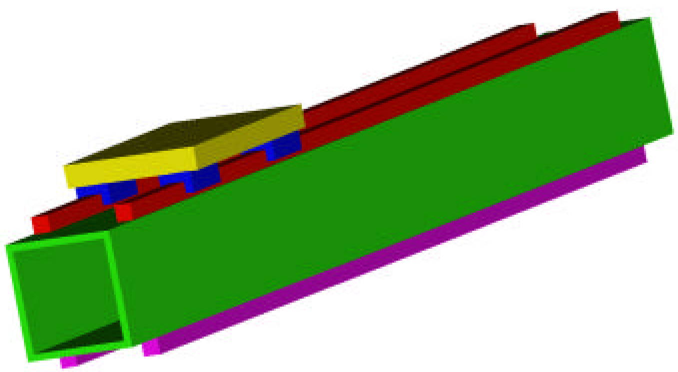




Before mounting | Back-to-back mounting after inner rings are clamped together | Before mounting | Face-to-face mounting after outer rings are clamped together

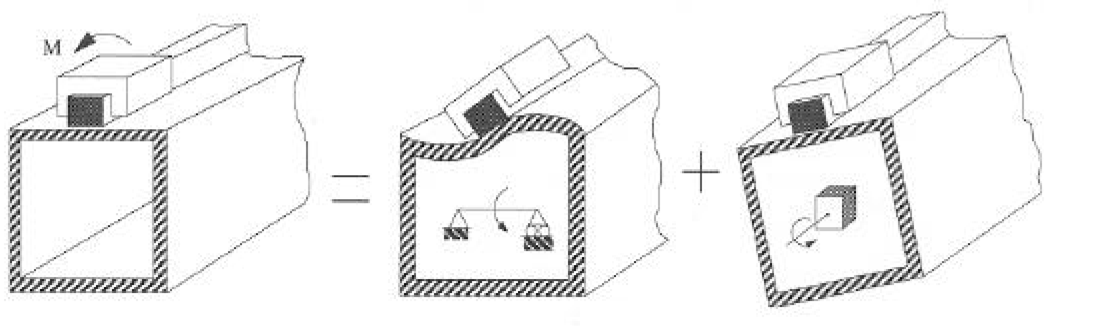
Symmetry

- Symmetry can be a powerful design tool to minimize errors
  - Thermal gradient errors caused by bi-material structures can minimize warping errors
    - Steel rails can be attached to the aluminum structure on the plane of the neutral axis
      - OR, Steel rails on an aluminum structure can be balanced by steel bolted to the opposite side
  - Angular error motions can be reduced by symmetric support of elements
- Symmetry can be detrimental (Maxwell applied to symmetry)
  - Differential temperature minimized by adding a heat source can cause the entire structure to heat up
    - Only attempt with extreme care
    - Better to isolate the heat source, temperature control it, use thermal breaks, and insulate the structure
  - A long shaft axially restrained by bearings at both ends can buckle
  - Remember-when you generalize, you are often wrong
    - The question to ask, therefore, is "Can symmetry help or hurt this design?"



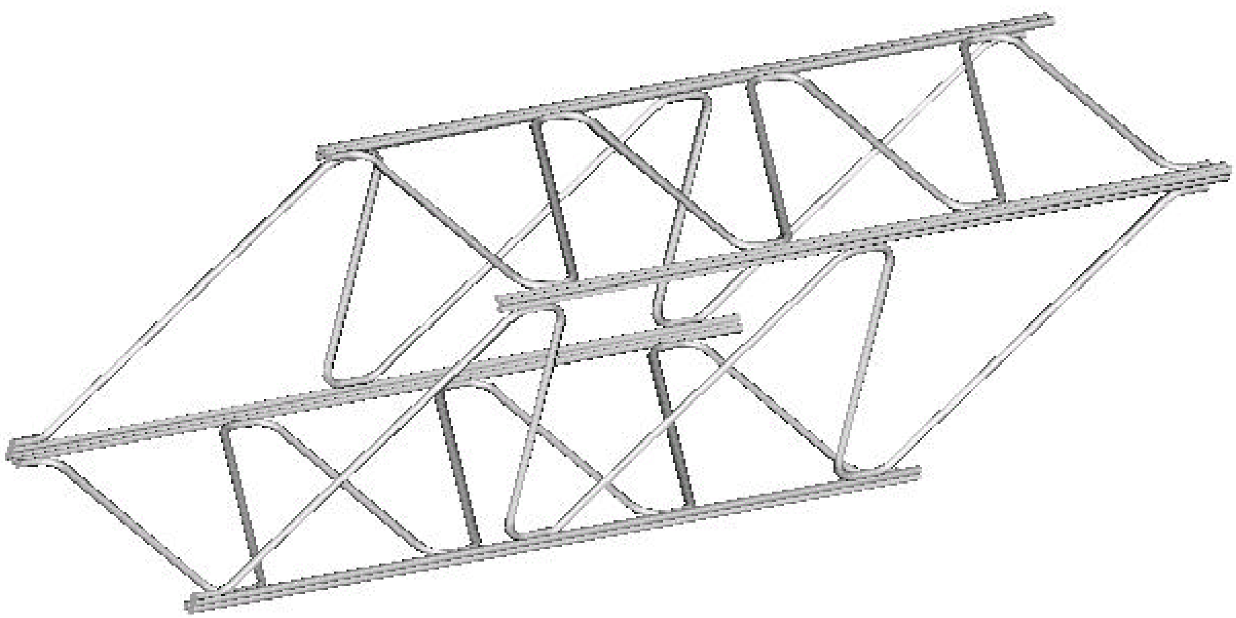
Superposition

- *With certain exceptions (large or plastic deformations), the effect (stress, strain, or deflection) produced on an elastic system by any final state of loading is the same whether the forces that constitute the loading are applied simultaneously or in any given sequence and is the result of the effects that the several forces would produce if each acted singly* (from R. Roark, W. Young, *Formulas for Stress and Strain,* 1975, McGraw Hill, New York)
- Even the most complex problems can be broken up into little manageable ones
  - Reciprocity also says little manageable problems can add up into big difficult problems if you are not careful!
- The key is to logically divide them up!



Parallel Axis Theorem

- The Parallel Axis Theorem is useful for calculating the moments of inertia for complex objects The Parallel Axis Theorem is even MORE useful for as a philosophy for design:
  - The stiffness of an design goes with the square of the distance of the structural members from the neutral axis (center)
    - Statics
    - Dynamics
    - Trusses



Parallel Axis Theorem: *Statics*

- The parallel axis theorem can be used to evaluate any cross section's inertia:

$$I = \sum_{i=1}^{N} I_i + \sum_{i=1}^{N} y_i^2 A_i$$

  - where y_i is the distance from the section nuetral axis to the center of each area of the section!
- The neutral axis is located a distance y_{NA} from an arbitrary plane:

$$y_{NA} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i}$$

- Most important: The PAT tells us that a section stiffens with the square of the distance from the neutral axis!
- E.g., when designing a laminate (1.5 mm AL sheet separated by wood core), double the core thickness and quadruple the panel stiffness!

Parallel Axis Theorem: *Trusses*

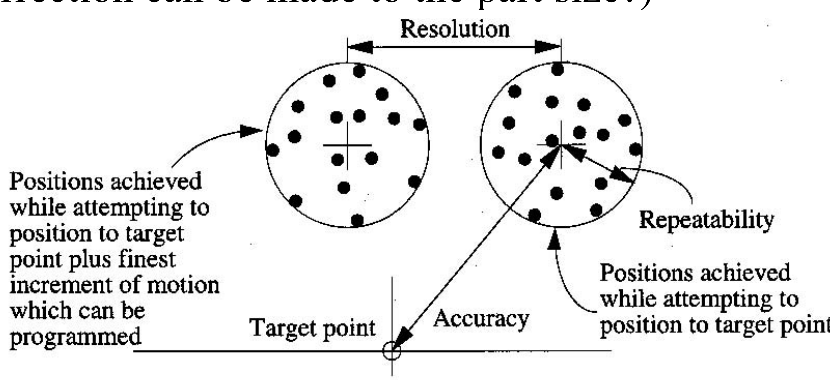
- A truss' strength can be estimated using I and I/c calculated using the parallel axis theorem.
  - The strength really only depends on the top and bottom chords!
  - For rod diameter D, truss height H, the effective moment of inertia for determining the strength of the truss is:

$$\frac{I}{c} = \frac{\pi D^2 H}{2}$$

  - Check for buckling based on Force = area*Mc/I
- John Mcbean took this to the extreme!

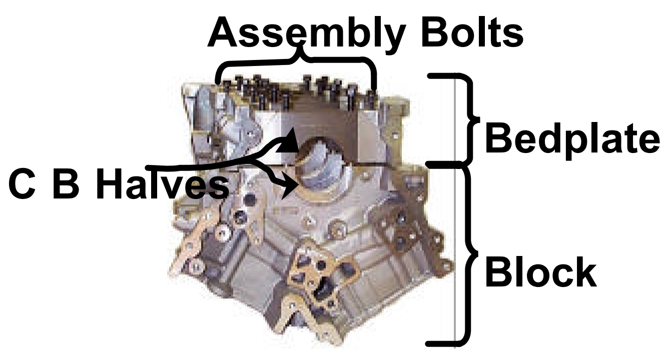
Accuracy, Repeatability, & Resolution

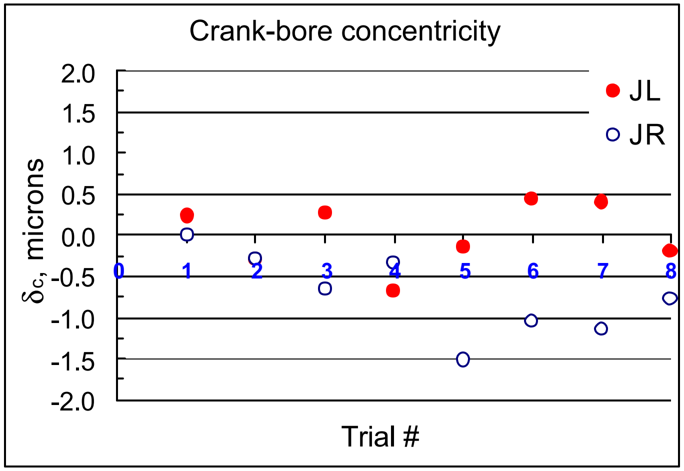
- Anything you design and manufacture is made from parts
  - Parts must have the desired accuracy, and their manufacture has to be repeatable
- Accuracy is the ability to tell the truth
  - Can the machine move exactly like you want it to?
  - Are the parts the exact size shown on the drawing?
- Repeatability is the ability to tell the same story each time
  - Can the machine at least make the exact same motion each time?
  - Are the parts all the same size?
- Resolution is the detail to which you tell a story
  - How well can you adjust the motion of the machine?
  - How fine a correction can be made to the part size?)



Accuracy, Repeatability, & Resolution: *Components*

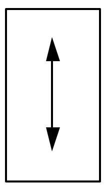
- Always ask yourself when designing something:
  - "Can the system be made with the desired accuracy?"
    - E.g., machine tool components must be straight, square
  - "Can the components of the system be made so they assemble accurately and/or repeatably??"
    - E.g., engine components must bolt together, be machined, be taken apart, and then assembled to fit back together exactly



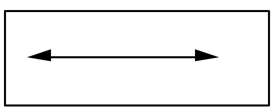


Accuracy, Repeatability, & Resolution: *Motions*

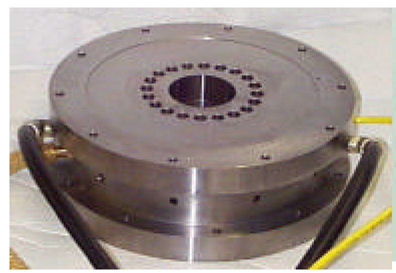
- It is often most important to obtain mechanical repeatability, because accuracy can often be obtained by the sensor and control system
  - When the error motions of a machine are mapped, the controller multiplies the part height by the axis' pitch & roll to yield the Abbe error for which orthogonal axes must compensate

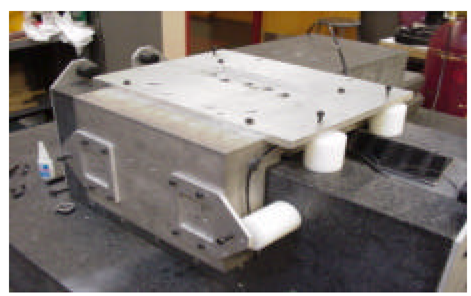


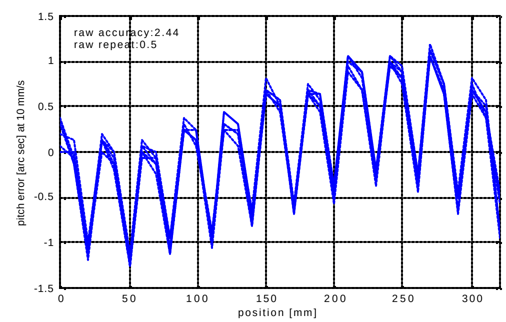
Y axis: Can be used to compensate for straightness errors in the X axis.



X axis: Can be used to compensate for straightness errors in the Y axis.



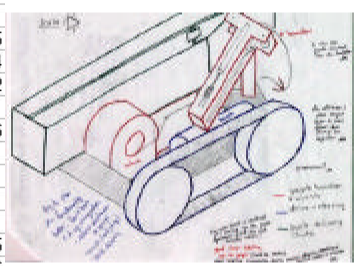


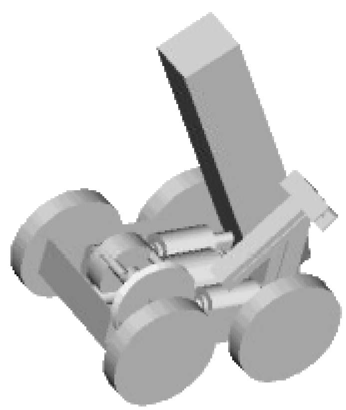


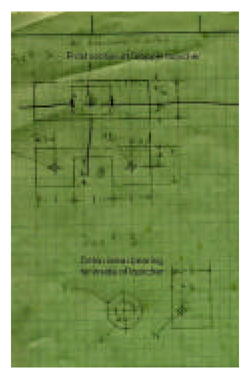
Accuracy, Repeatability, & Resolution: *Philosophy*

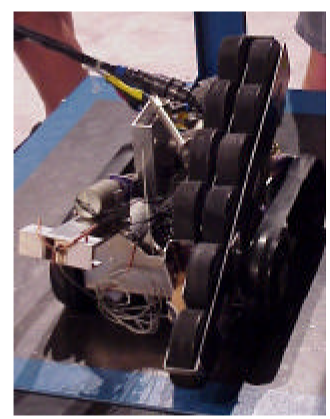
- Accuracy, Repeatability, & Resolution *are not just terms for mechanical performance, they are also terms for how to execute the design process!*
  - *My machine performed very well in the contest. It had zero malfunctions and we managed to* ***hit dead on with the grapple hook every time****. In round 4, we introduced the trailer addition to the machine. Everyone was surprised that someone could score more than 50 points. I got the highest score of the night at 58 points and went on to win the contest. Prof. Slocum seemed pretty excited that I had broken 50 points. Here he is picking me up and spinning me around in circles.*
    David Arguelles Winner of 1999 ***MechEverest*** Contest
    - http://web.mit.edu/darguell/portfolio/2.007
    - The winner of the contest gets an optional celebrational ceremonial ride around the contest table, courtesy of Prof. Slocum

Hook launcher Model	
weight of hook (Kg)	0.05
muzzle velocity	9.4
Number of springs	2
d (draw)	0.095216
Winch model	
radius	0.05
mass	6
w (rpm)	55
torque	2.1
velocity	0.287833



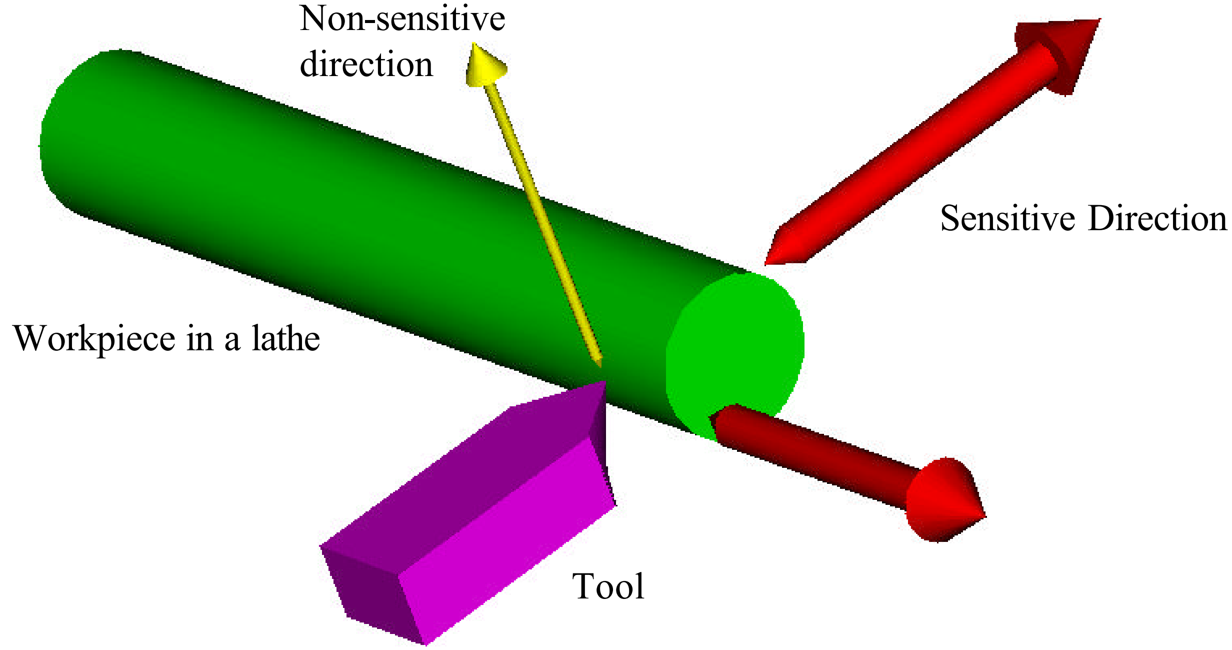






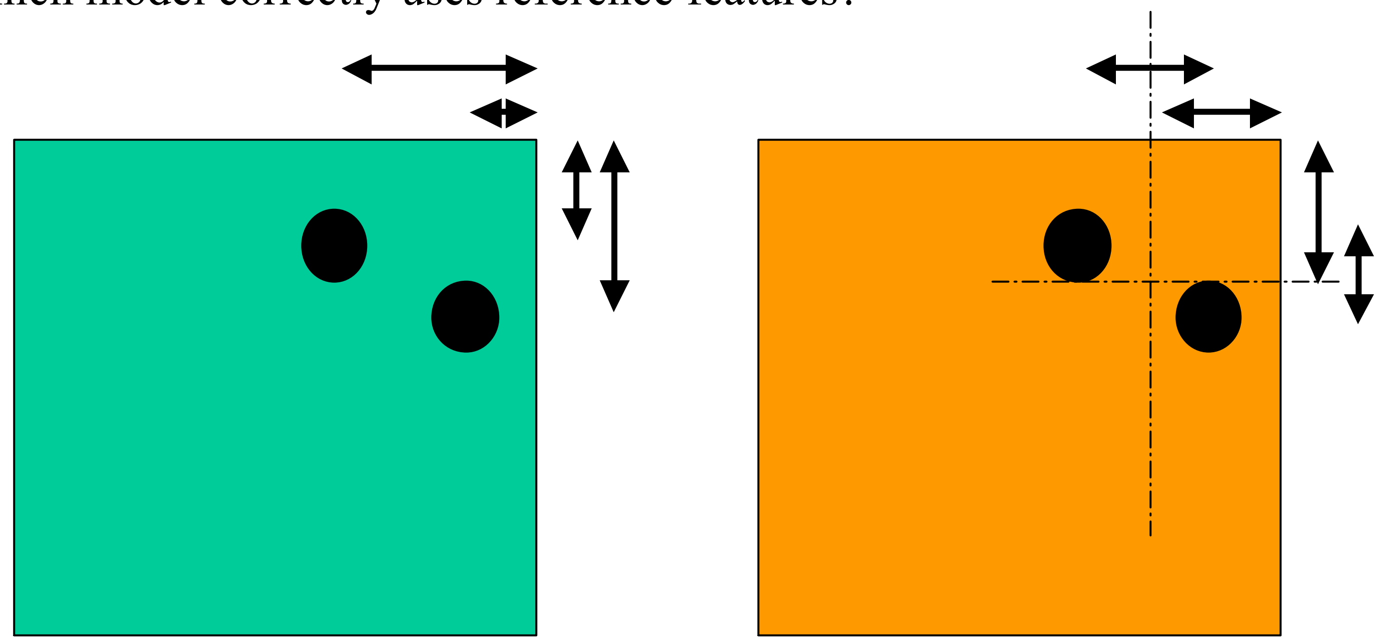
Sensitive Directions

- In addition to Accuracy, repeatability, and resolution, we have to ask ourselves, "when is an error really important anyway?"
  - Put a lot of effort into accuracy for the directions in which you need it
    - The *Sensitive Directions*
    - Always be careful to think about where you need precision!

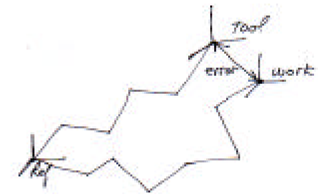


Reference Features

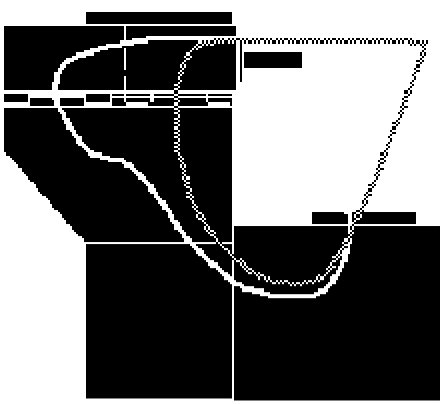
- Robust designs have *Reference Features* from which measurements are made with respect to, and to which components are attached
  - Anybody can create a solid model, but will the solid model still work if features or components are deleted?
  - Anybody can make one of something, but making things interchangeable requires that they be systematically measured…
- Which model correctly uses reference features?

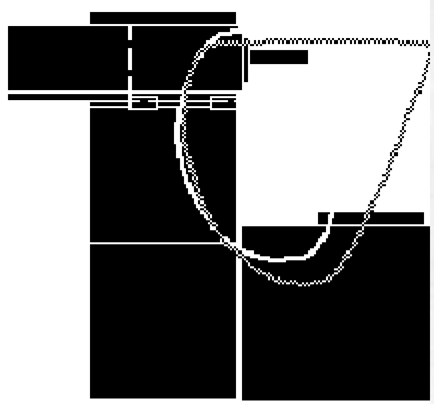


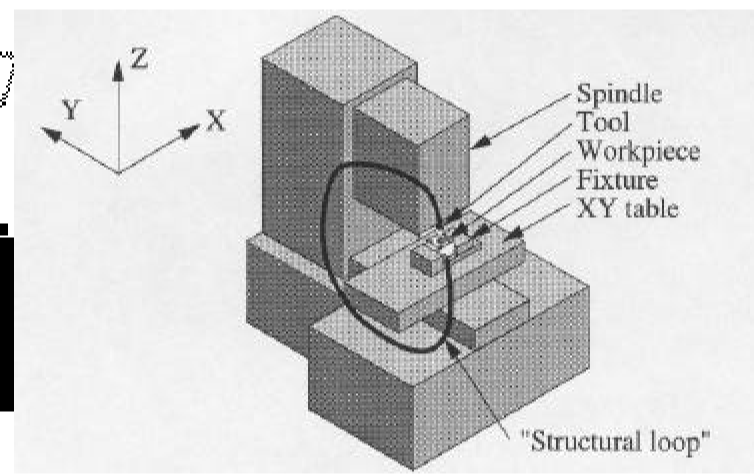
Structural Loop



- The Structural Loop is the path that a load takes as it travels from the tool through the structure and to the work
  - It contains all the joints and structural elements that position the tool with respect to the workpiece
  - It can be represented as a stick-figure to enable a design engineer to create a concept
  - The accuracy and load capacity of a machine depends on the structure and machine elements the design engineer select
  - Subtle differences can have a HUGE effect on the performance of a machine
  - The structural loop gives an indication of machine stiffness and accuracy
    - Long-open loops have less stiffness and less accuracy

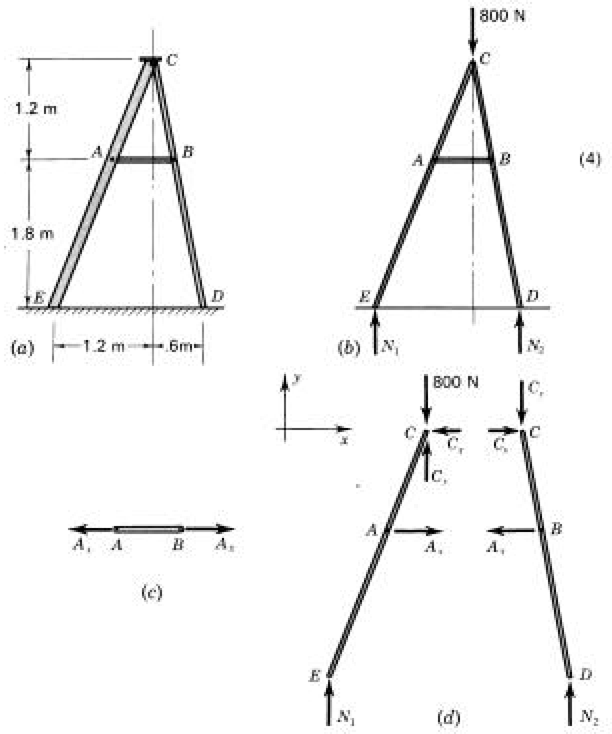






Free Body Diagrams

- Free body diagrams allow a designer to show components and their relationship to each other with respect to forces transmitted between them
  - Invaluable for properly visualizing loads on components
  - In order to properly constrain a component, one has to understand how it is loaded and constrained
  - Example: FBD of a ladder and components (Crandall, Dahl, Lardner)

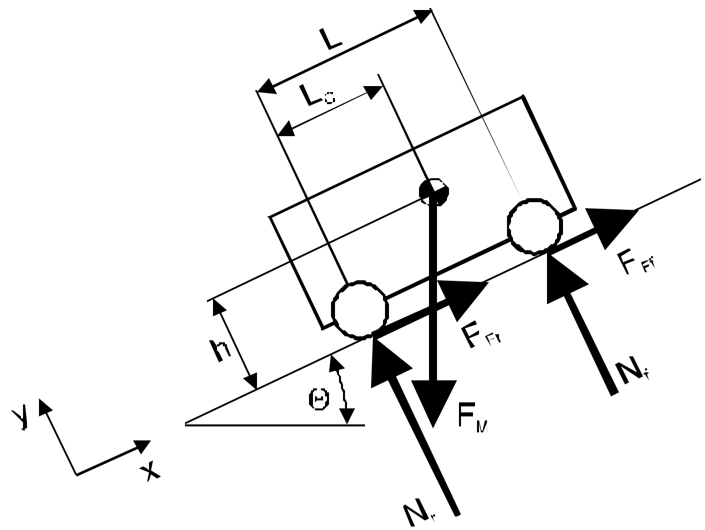


Centers of Action

- The *Centers of Action* are points at which when a force is applied, no moments are created:
  - Center of Mass
  - Center of Stiffness
  - Center of Friction
- A system is most robust when forces are applied as near as possible to the *Centers of Action*

Centers of Action: *Center of Mass*

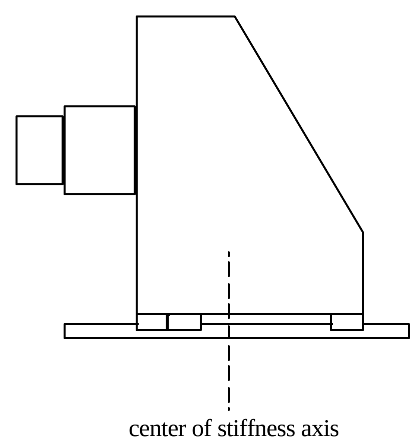
- The concept of the Center of Mass (or center of gravity, cg) is well known to most people
  - The center of mass of a system of particles moves like a single particle of mass $M=\Sigma m_i$ under the influence of the resultant external force acting on the system
    - The center of mass is the point at which when a force is applied, an object undergoes only linear acceleration and thus has no angular acceleration component (which would otherwise lead to *Abbe* errors!)
- When a vehicle drives up an incline, it will not tip over if the downward projection of the cg remains within the wheelbase *(more on this in the lecture on Robustness)*



Centers of Action: *Center of Stiffness*

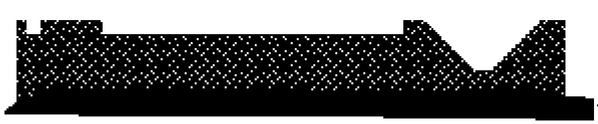
- A body supported by bearings, behaves as if all the bearings are concentrated at the ***center of stiffness***
  – The point at which when a force is applied to a locked-in-place axis, no angular motion of the structure occurs (which would otherwise lead to *Abbe* errors!)
  – It is also the point about which angular motion occurs when forces are applied elsewhere on the body
    - EXTREMELY POWERFUL PHILOSOPHICAL TOOL, AS IT ALLOWS A DESIGNER TO CREATE A MACHINE AS A STICK FIGURE
    - Structure and bearings are added during embodiment phase so as to make the center-of-stiffness at the stick figure nodes
  – Found using a center-of-mass type of calculation (K is substituted for M)

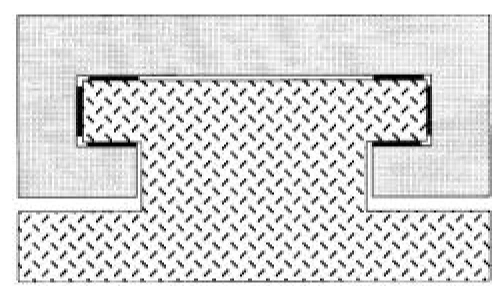
$$X_{center\_of\_stiffness} = \frac{\sum_{i=1}^{N} X_i K_i}{\sum_{i=1}^{N} X_i}$$



Centers of Action: *Center of Friction*

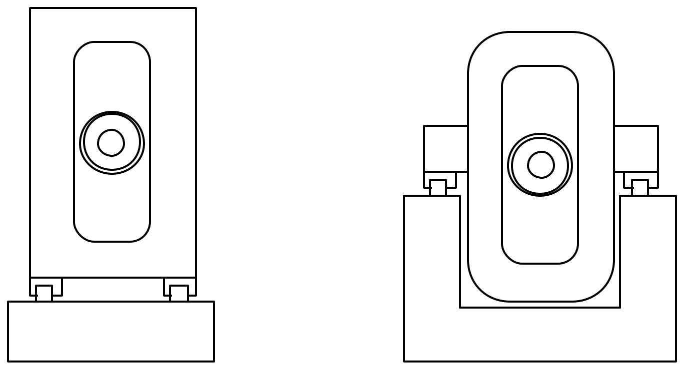
- A body supported by bearings, has friction forces in proportion to the weight distribution and the number and position of the bearings
- The ***center of friction*** is the point at which when a force is applied to a moving structure to which no other external forces are applied, no angular motion of the structure occurs (which would otherwise lead to *Abbe* errors!)
  - Found using force and moment balance equations that consider the effects of friction, bearing geometry, and center of gravity
    - The center of friction is sometimes, but not always, located at the center of stiffness
      - If a load is applied to different positions on a Vee-and-Flat and a Double-Vee supported carriage, how do the center of friction and the center of stiffness vary?
        - » How do they compare to a boxway design?
        - » What are the cost/benefits of the two designs?
        - » What are the manufacturing issues?



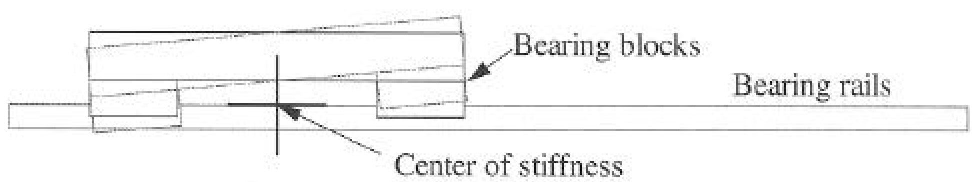


Centers of Action: *Robust Design*

- If the primary work zone is near the center of mass, stiffness, and friction, errors will be minimized
- If the actuators for a machine apply their forces near the center of mass, stiffness, and friction, errors will be minimized
  - It is often difficult to make all three coincide, but get them as close as you can

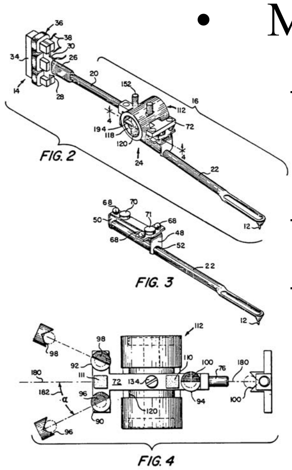


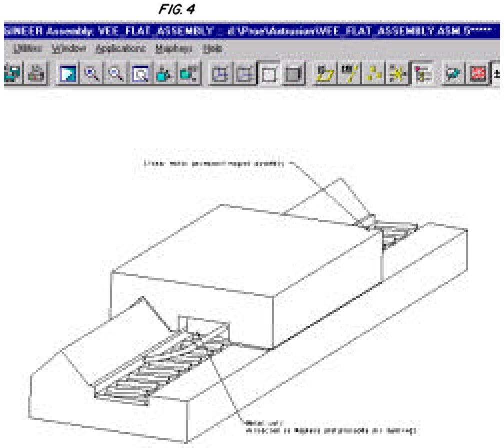
- If a machine element (e.g., a leadscrew nut) is located at the center of stiffness, then error motions of one machine element (wobble of the screw) will not cause pitch errors (Abbe errors) in another element (carriage)

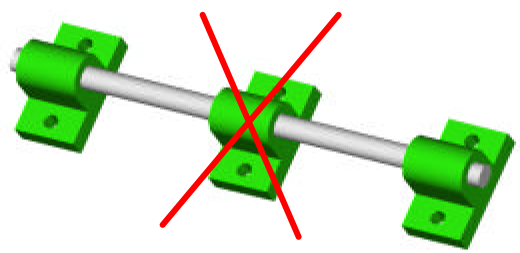


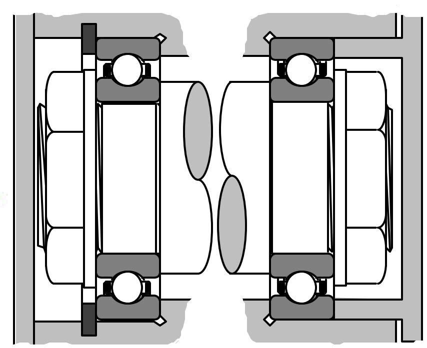
Fundamental Principles: *Exact Constraint Design*

- Every rigid body has 6 Degrees of Freedom
  - An exactly constrained design has no chance of deforming or having its function impaired when after the components are assembled, be it by fastener tightening or thermal expansion
- Make sure you have constrained what you want to constrain!
  - If the body is to have N degrees of freedom free to move, there has to be 6(N-1) bearing reaction points!
  - To resist translation, a force is required.
  - To resist rotation, a moment, or two forces acting as a couple, is required!
    - Saint-Venant rules! Do not constrain a shaft with more than 2 bearings, unless it is very long…

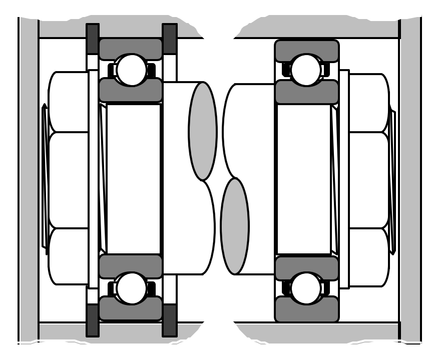








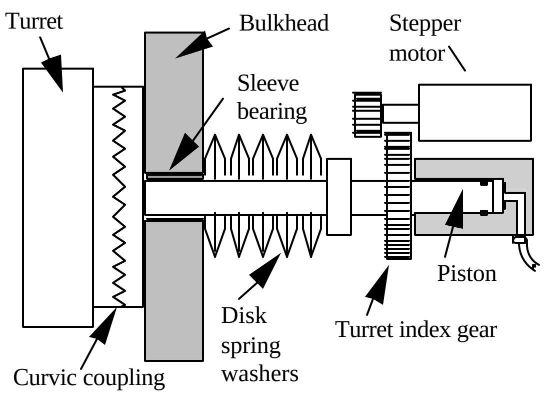
Overconstrained

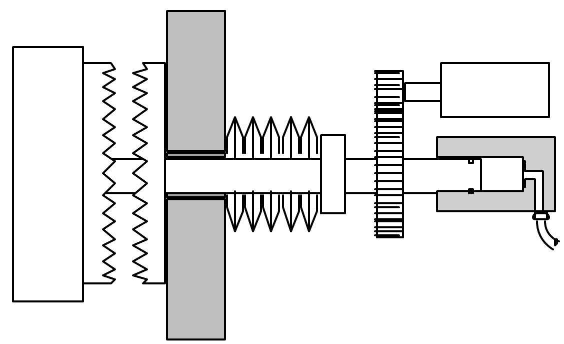


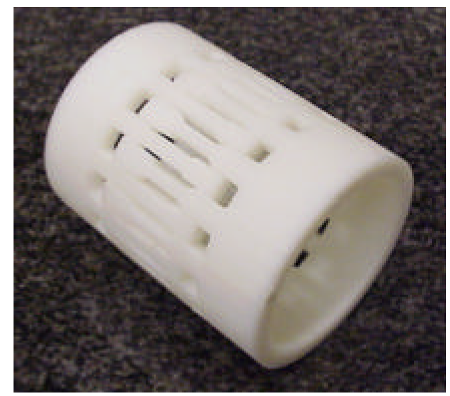
Properly constrained

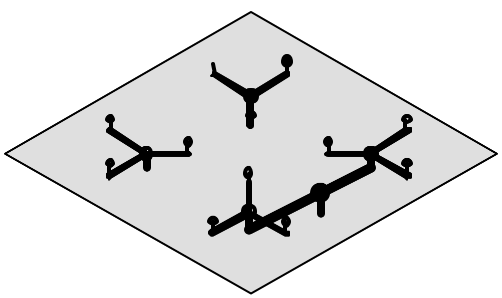
Fundamental Principles: *Elastic Averaging*

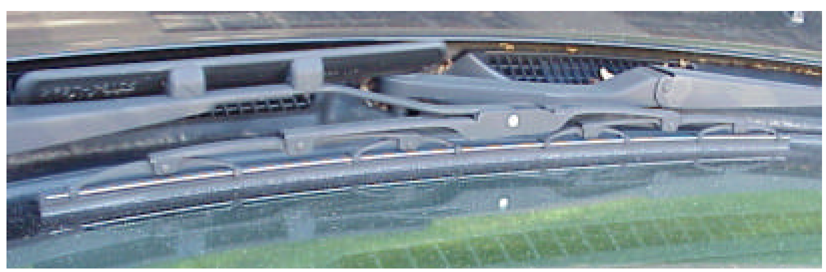
- Applying Reciprocity to Exact Constraint Design implies that instead of having an exact number of constraints, have an "infinite" number of constraints, so the error in any one will be averaged out!
  - Indexing tables often use a *curvic* coupling (two face gears meshed together) to achieve a high degree of elastic averaging (accuracy), stiffness, and load capacity
  - Windshield wiper blades and surface plate or large-mirror supports distribute the loads using *wiffle trees*

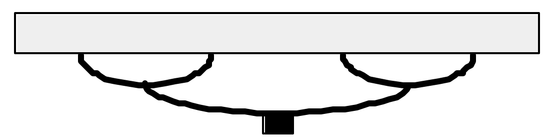










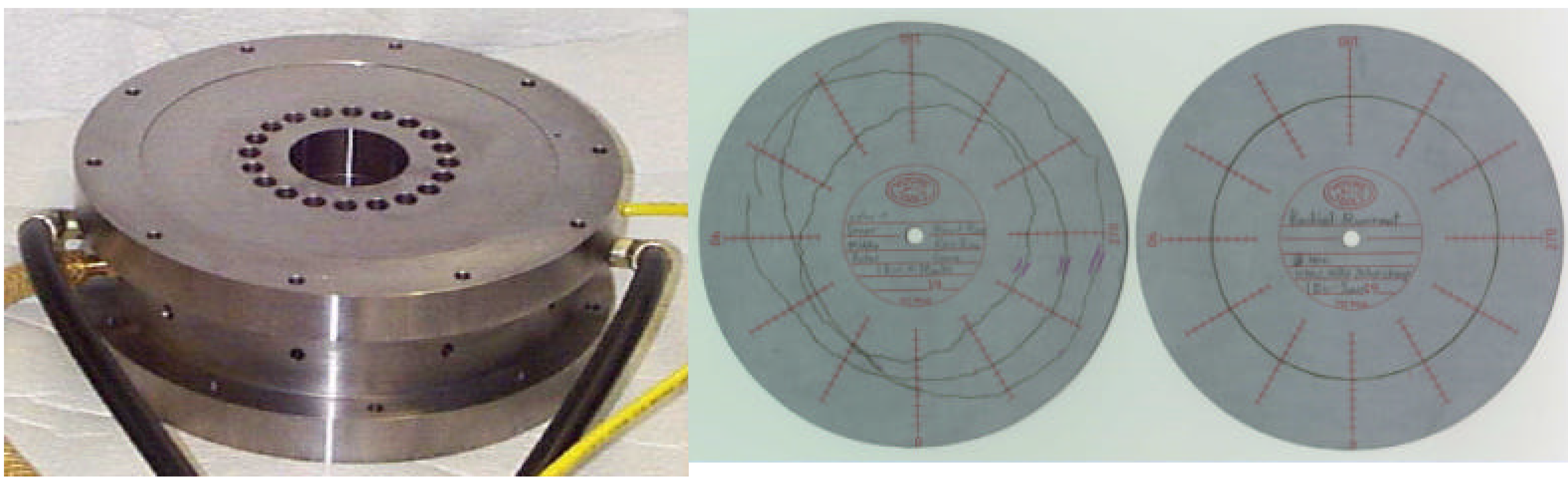


Elastic Averaging: *Error Management*

“Random results are the result of random procedures”

Geoffe Portes, Cranfield Unit for Precision Engineering (CUPE)

How is something made from parts accurate to 5 microns, accurate as an assembly to 0.05 microns?



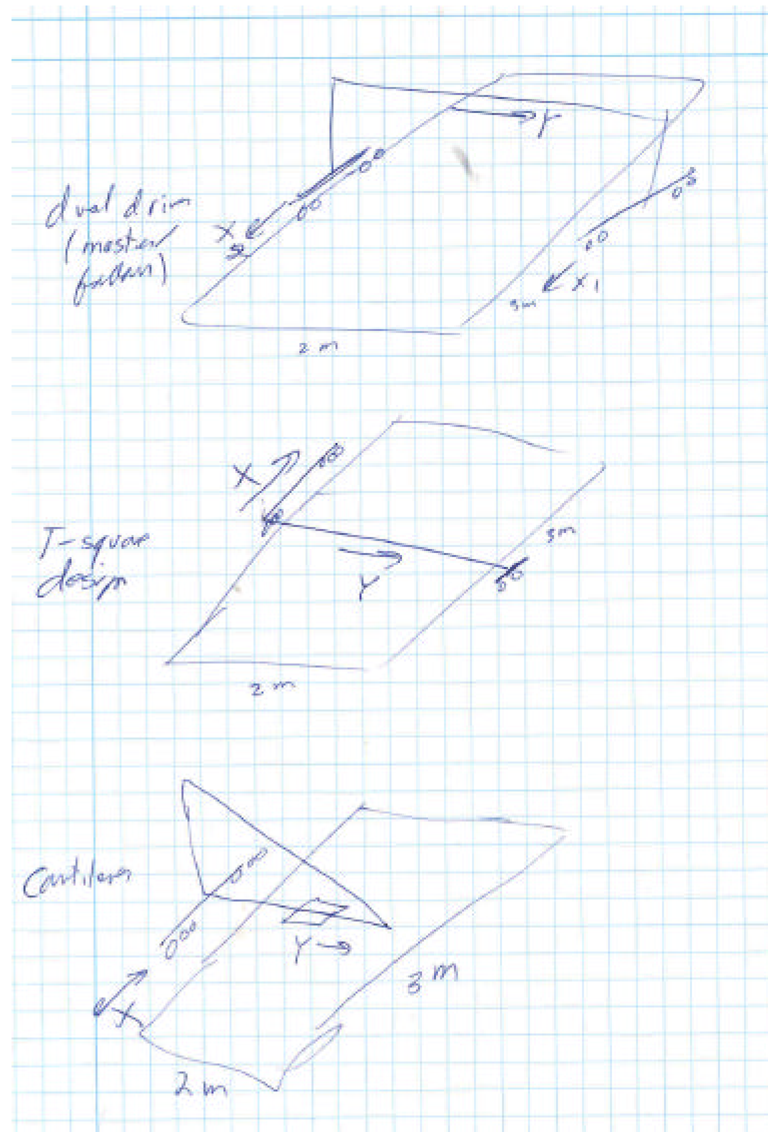
Rotary Table with 50 nanometer radial error motion designed by Prof. Slocum’s graduate students Nathan Kane and Joachim Sihler

Fundamental Principles: *Dimensional Analysis*

- *Buckingham-Pi theorem:* The units must work out!
  - You can often "derive the formulas" just by making the units work!
  - You will at worst be off by a constant, but will at least get the trends correct!
- ALWAYS check the units on your calculations (e.g., stress =N/m^2)
  - You can often use dimensional analysis to help you identify physical parameters that must be in the analysis
- POWER &/or WORK &/or ENERGY calculations often provide the most insight as to what are the sensitive parameters of a problem
  - Power = Force*Velocity
  - Work = Force*Distance
  - Energy = $^1/_2 mV^2$
  - Energy = $^1/_2 kx^2$

Fundamental Principles: *Stick Figures*

- Applying ALL the above principles allows a designer to sketch a machine and error motions and coordinate systems in terms of a *stick figure* only
  - The sticks join at centers of stiffness, mass, friction, and helps to:
    - Define the sensitive directions in a machine
    - Locate coordinate systems
  - Set the stage for error budgeting
  - The designer is no longer encumbered by cross section size or bearing size
    - It helps to prevent the designer from locking in on a shape too early
- Error budget and preliminary load analysis can then indicate the required stiffness/load capacity required for each "stick" and "joint"
  - Appropriate cross sections and bearings can then be deterministically selected
- It is a "backwards tasking" solution method that is very very powerful!



Conclusion: *Fundamental Principles Guide Designs*

- Before any analysis is done, an engineer has to have a vision in mind
  - The proportions of the vision are guided by the engineer's bio neural net
  - The efficiency of the bio neural net is directly governed by the depth of understanding of fundamental principles
    - Fundamental principles are best learned by experimentation!