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2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

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2.830J/6.780J Control of Manufacturing Processes

Spring 2008

Assignment 8

Out 4/10/2008; due **Friday** 4/18/2008

It would be helpful to read this before the lecture on nested variance on Tuesday, April 15th. The reading is entitled *Variance Components and Process Sampling Design* (Chapter 3 of *Statistical Methods for Industrial Process Control* by David Drain). An accompanying spreadsheet has also been posted.

Problem 1

Here we deal with the data set provided in the Excel spreadsheet 8-1.xls. This file contains 31 (x, y) measurement pairs and it is believed that the underlying physical process can be modeled as $y = b_0 + bx + \varepsilon$.

- Fit a linear model to the data, following the procedure described in May and Spanos section 8.1.2.3. Produce a plot containing the raw data, the straight-line model fit, and a 99% confidence interval for the model. Please show your working.
- Do you consider any of the data points to be obvious ‘outliers’? Can you think of systematic ways of identifying and discounting outliers during a fitting procedure? Here we consider the possibility of using the confidence interval as a way of automatically rejecting outliers. Reduce the dataset to only those data contained within the 99% confidence interval found above. Fit a new linear model to this reduced dataset, and on a new graph plot the reduced dataset, the new linear model and the corresponding 99% confidence interval. Comment on differences between your best estimates of b and b_0 in parts (a) and (b).
- Reduce the dataset once more, by rejecting data points lying outside the new confidence interval determined in part (b) above. What do you notice about the resulting best estimates of b and b_0 compared to the previous estimates? What would happen if you attempted to continue this cycle of data rejection and fitting? Does this approach to rejecting outliers seem sensible?

Problem 2

Suppose that we fit the following model to some data:

$$\hat{y} = b_0 + b_1x_1 + b_{11}x_1^2 + b_2x_2$$

Derive an expression for a $(1 - \alpha)$ confidence interval for \hat{y}^* , the model prediction at a given point (x_1^*, x_2^*) . Use May and Spanos section 8.1 to help you.

Problem 3

Problem 4 on pp. 222–223 of Drain.

Problem 4

In lecture 16 the concept of robustness was introduced by considering two types of factors: control and noise. The latter were defined as factors we could change in a controlled experiment, but which would be random during normal process operations. Robustness was then defined as finding an operating point for the control factors that would minimize the effect of these noise factors while still giving good process capability.

- (a) Two methods were put forward: use of inner and outer arrays and a response surface method to find mean and variance response surfaces. Please compare these two methods with respect to the number of experiments necessary to find an optimum operating point. For each method, express the number of experiments needed as a function of the number, c , of control factors, and the number, n , of noise factors.

Now consider a slightly different problem. There are only control factors and noise. The latter are not factors, nor can we alter them during an experiment. However you still want to see if some type of process robustness study can be performed.

- (b) Using a basic response model (i.e. $y = b_0 + b_1x_1 + \dots + \varepsilon$) state the assumptions under which such an optimization would be possible. (Recall that we are trying to minimize the sensitivity of the process to parameter variations.)
- (c) How would you set up an experiment to conduct this “pure noise” robustness test, and what methods would you use to test the assumption(s) from part (b)?