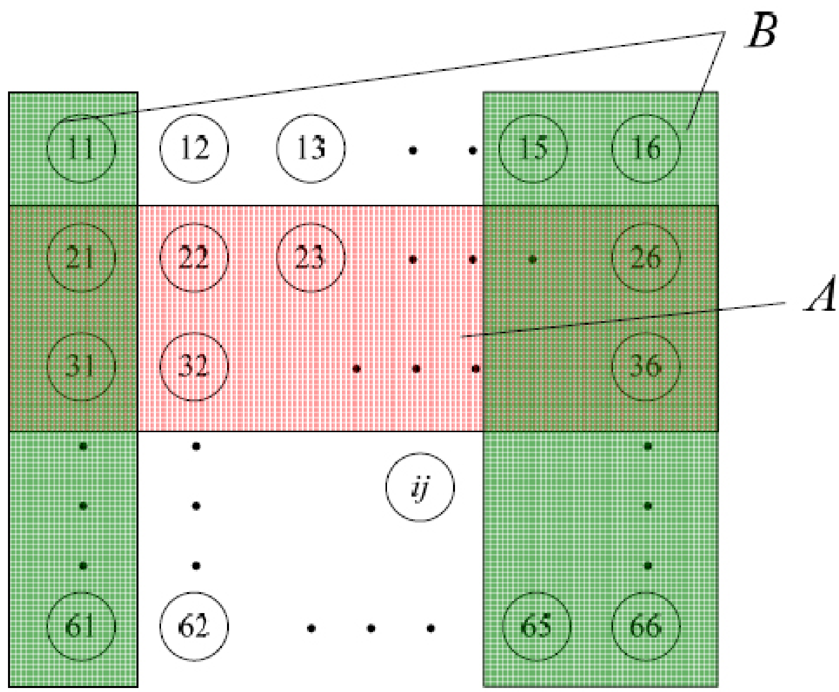
Notes for Lecture 3

Example of Independence



$$A = \{i = 2 \ or \ 3\};$$

$$B = \{j = 1 \ or \ 5 \ or \ 6\}.$$

Thus, we have

$$A \cap B = \{(2,1),(3,1),(2,5),(3,5),(2,6),(3,6)\}.$$

So, we can compute the following:

$$P(A) = 12/36 = 1/3;$$

$$P(B) = 18/36 = 1/2;$$

$$P(A \cap B) = 6/36 = 1/6 = P(A)P(B).$$

We can also demonstrate the independence in the following way.

Let

$$prob(ij) = f(i)g(j)$$

Thus,

$$
\begin{aligned}
prob(A) &= f(2)g(1) + f(2)g(2) + \cdots + f(2)g(6) \\
&\quad + f(3)g(1) + f(3)g(2) + \cdots + f(3)g(6) \\
&= (f(2) + f(3))(g(1) + g(2) + \cdots + g(6))
\end{aligned}
$$

Similarly, we have

$$prob(B) = (g(1) + g(5) + g(6))(f(1) + f(2) + \cdots + f(6))$$

And

$$prob(A \cap B) = f(2)g(1) + f(3)g(1) + f(2)g(5) + f(3)g(5) + f(2)g(6) + f(3)g(6)$$

Note that

$$f(1) + f(2) + \cdots + f(6) = 1$$

And

$$g(1) + g(2) + \cdots + g(6) = 1$$

Therefore,

$$prob(A) = f(2) + f(3)$$

$$prob(B) = g(1) + g(5) + g(6)$$

Thus,

$$
\begin{aligned}
prob(A)\,prob(B) &= (f(2) + f(3))(g(1) + g(5) + g(6)) \\
&= f(2)g(1) + f(3)g(1) + f(2)g(5) + f(3)g(5) + f(2)g(6) + f(3)g(6) \\
&= prob(A \cap B)
\end{aligned}
$$

So, A and B are independent.

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