

Variation in Assemblies

- Goals of this class
 - Understand how to represent variation at the assembly level
 - Link models to models of variation at the part level
 - Link models to nominal chain of frames
 - Get an idea of statistical process control and statistical tolerancing vs worst case tolerancing

History of Tolerances in Assemblies

- Statistical process control and statistical tolerancing permit a bet on interchangeability
- Coordination makes a comeback as functional build
- Quality today is more than interchangeability
 - The product almost certainly will “work”
 - Real quality means
 - Durability
 - Reliability
 - Low noise, vibration, etc
 - All are associated with low running clearances, fine balance, etc., all requiring closer tolerances, robust design, etc.

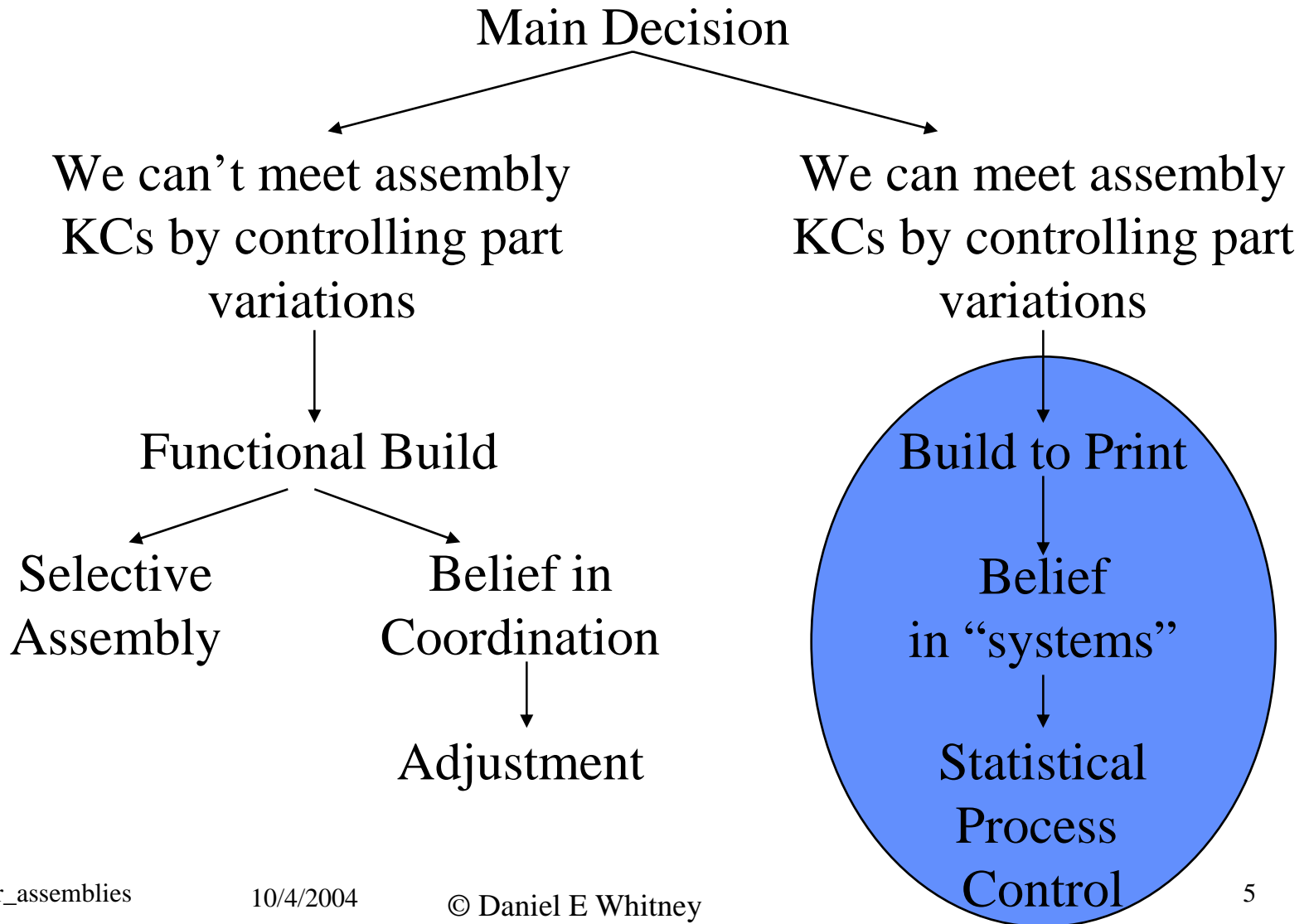
Recent Gains in Productivity

- Wall Street Journal, Sept 28, 00, p 2:
 - “Many analysts may have overlooked one of the main drivers of the productivity gains of the last five years: machine tools... more than half the increase has occurred outside the computer, software, and telecommunications sectors...
 - Greater precision in machine tools has helped cut the energy use of air conditioners by 10% between 1990 and 1997 ... using a new kind of compressor whose manufacture required ... precision down to 10 microns”
 - One of the principal reasons for more reliable automobile transmissions ...
 - Open architecture, digital programming, ceramic tool bits are part of the picture

KCs and Variation

- A KC is delivered when it achieves its desired value within some specified tolerance
- This puts the emphasis on behavior at the assembly level, not the part level
- To deliver the KC we must
 - Design the chain and the location strategy at each joint in the chain
 - Ensure that variation at the part level does not propagate to the assembly level and upset delivery of the KC
 - This means understanding variation in each of the joints in the chain

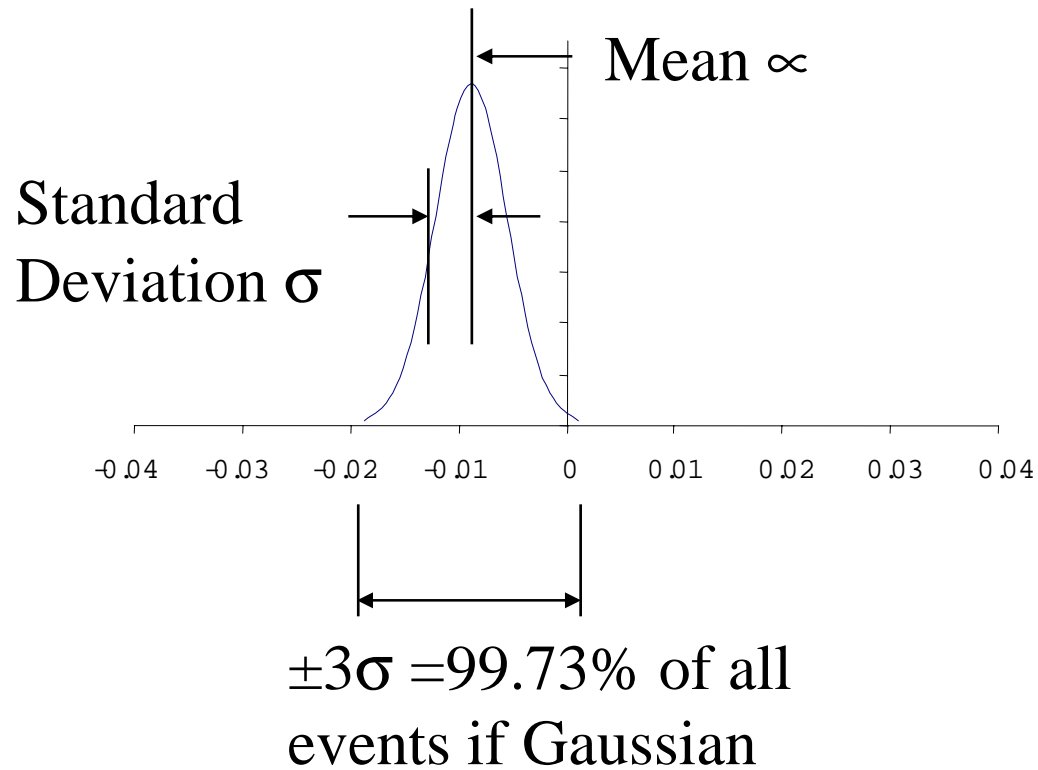
Logic Tree of Control of Variation



Two Kinds of Errors

- Change in process average (mean shift)
- Process variation around the mean (variance)
- These are different kinds of errors that require different approaches
 - The screwdriver workstation-RCC story
 - Whitehead's reason for using kinematic design: to eliminate variation (“erratic errors”)
 - Taguchi's systematic approach
 - Correct mean shift first, then reduce variation
 - The other sequence makes sense, too

Basic Definitions



Precision vs Accuracy

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Source:

Figure 5-16 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Consistent but
consistently wrong

Right on the average

Best Situation

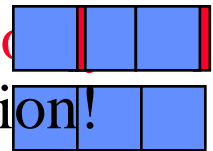
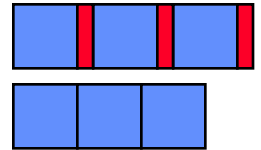
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Source:

Figure 5-16 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Models of How Errors Accumulate in Assemblies

- Worst case tolerancing essentially tracks the max error, treats it as mean shift, and ignores the variance
 - Assumes all errors are at their extremes at the same time
 - This is deterministic, not statistical
 - Errors accumulate linearly with the number of parts
- Statistical tolerancing (AKA root sum square) tracks the variance and assumes zero mean shift
 - Assumes errors are distributed randomly \pm between limits
 - Errors accumulate with SQRT of number of parts but if the average part error = the desired nominal dimension! (i.e., average part (& assy) error = 0.)



Stapler Variations

TOP VIEW

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Source: Figure 5-14
in [Whitney 2004] Whitney, D. E.
*Mechanical Assemblies:
Their Design, Manufacture, and
Role in Product Development.*
New York, NY: Oxford University
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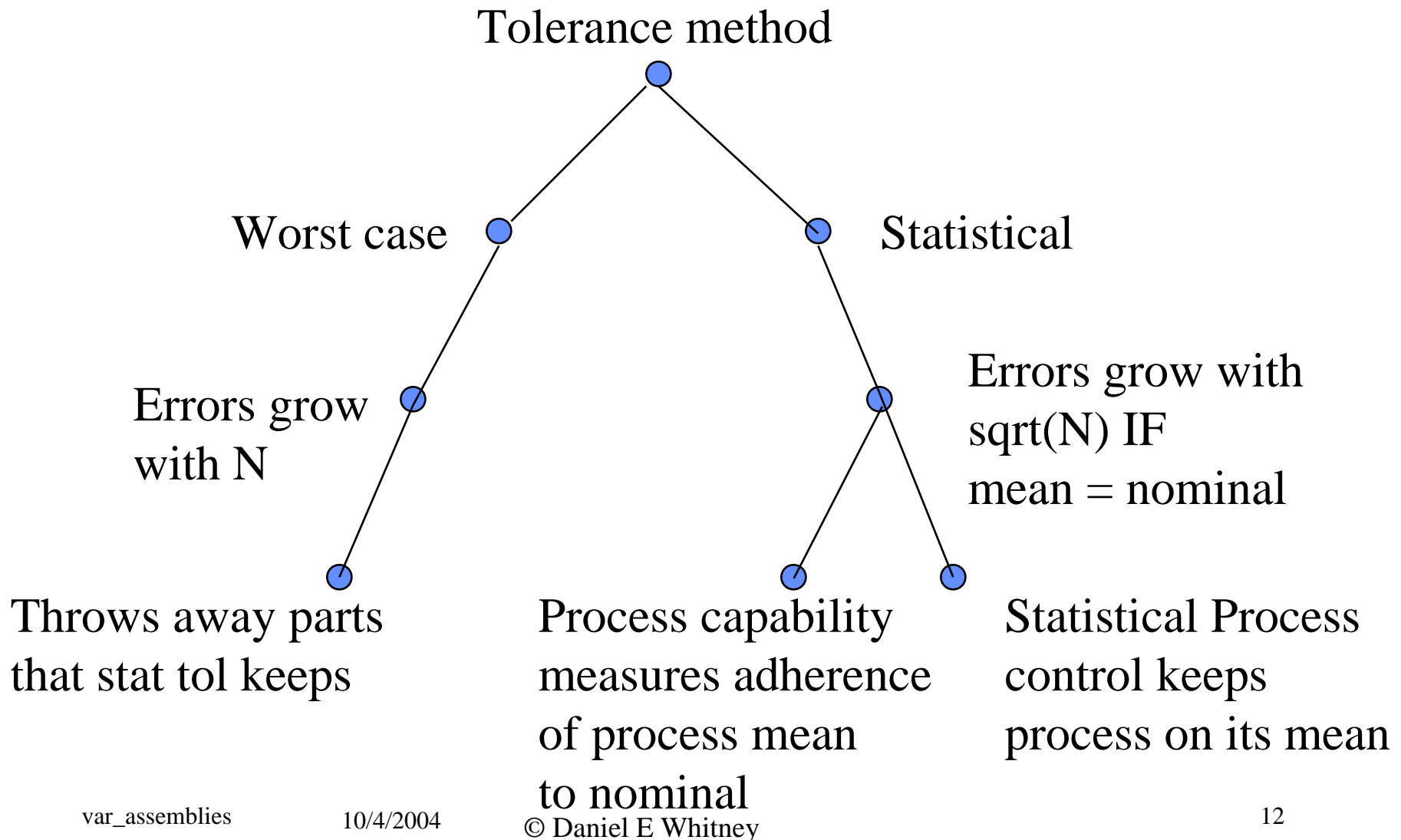
Source:
Figure 5-15 in [Whitney 2004] Whitney, D. E.
*Mechanical Assemblies: Their Design, Manufacture,
and Role in Product Development.*
New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

FRONT VIEW

Worst Case

Statistical

Logic Tree of Statistical and Worst-Case Tolerancing



How Variations Accumulate

- Assumptions

- All variations are described by the same distribution
- All variations are independent of each other
- The SSN jackpot in Milwaukee

- Then:

- The mean or average of a sum of such variations is the sum of their individual means

$$E\left[\sum_{i=1}^N x_i\right] = \sum_{i=1}^N E[x_i] = NE[x_i] \quad \text{if all } E[x_i] \text{ are the same}$$

- Actually this is true regardless of the above assumptions
- The variance of the sum of such variations is the sum of their individual variances

$$\sigma^2\left[\sum_{i=1}^N x_i\right] = \sum_{i=1}^N \sigma^2[x_i] = N\sigma^2[x_i]$$

Total Error

$$\begin{aligned} \text{Total Accumulated Error} &= \\ &\text{Accumulated Mean Errors} \\ &+ \text{Accumulated Standard Deviation Errors} \\ &= \sum_{i=1}^N \text{meanshift}_i + \sqrt{\sum_{i=1}^N (3\sigma_i)^2} \\ &= N * \text{meanshift} + \sqrt{N} * 3 * \text{stddev} \end{aligned}$$

if all meanshifts and standard deviations
are the same

What It Means

- Sums of **zero-mean** errors accumulate at the rate \sqrt{N} because the + and - errors tend to cancel
- Sums of **non-zero-mean** errors accumulate at the rate N because there are no cancellations
- Therefore, non-zero-mean errors accumulate **much more rapidly** than zero-mean errors

Root Sum Square (RSS) Method

Assume tolerance band = $\pm T$

Assume $T = 3\sigma$ of error distribution

Then accumulated error from N of $T_i =$

$$\text{Accumulated error} = \sqrt{\sum_{i=1}^N T_i^2}$$

But this is really the entire accumulated error only if there is no meanshift. If there is meanshift then the RSS method will badly under-estimate the error.

Why the Mean Should Seek the Nominal

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Source:

Figure 5-21 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

How to survive Las Vegas

Statistical and Worst Case Compared

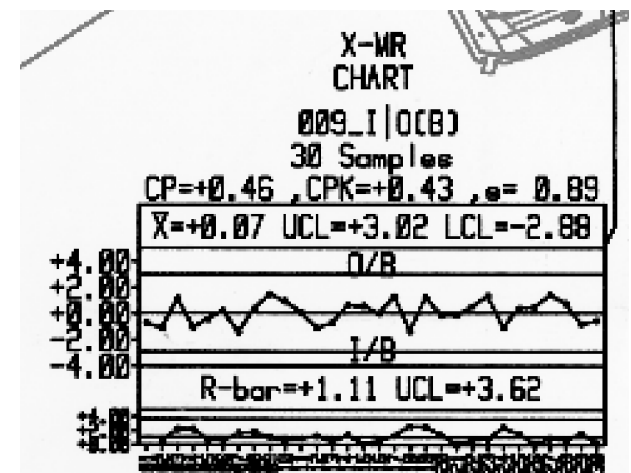
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Source:

Figure 5-22 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Statistical Process Control

- SPC aims to get a process in **statistical control**
 - The mean varies in a random way and stays within the mean control limits
 - The range varies in a random way and stays within the range control limits
- X-bar and R charts plot the mean and range of samples taken from production parts
- Deviations from the mean, consistent errors, and excess variation can be found quickly and eliminated before bad parts are made
- If a process is in control, then there is a chance that it can deliver parts without mean shift



Process Capability

- Every process has a mean and a “natural tolerance” range which should be tighter than the product needs
- Process capability index C_{pk} tracks whether the process range is within the 3σ tolerance band and the process mean is on the target nominal dimension

$$C_{pk} = \min \left[\frac{USL - \bar{X}}{3\sigma} ; \frac{LSL - \bar{X}}{3\sigma} \right]$$

USL = upper spec (tolerance) limit

LSL = lower spec (tolerance) limit

\bar{X} = mean of the process

σ = variance of the process

Process Capability Cpk

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Source:

Figure 5-18 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

# of σ	% within \pm # of σ	% outside
1	68.269%	31.731%
2	95.450%	4.550%
3	99.730%	0.270%
4	99.994%	0.006%

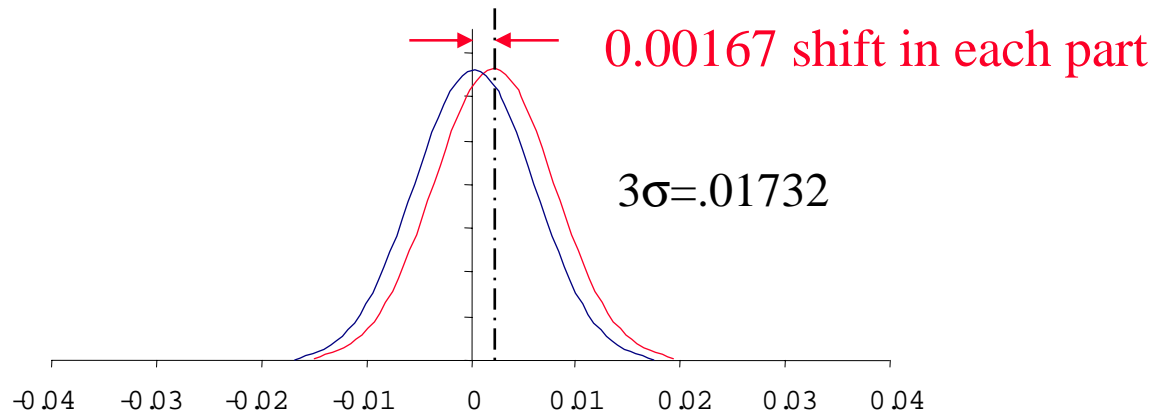
Process Control & Capability

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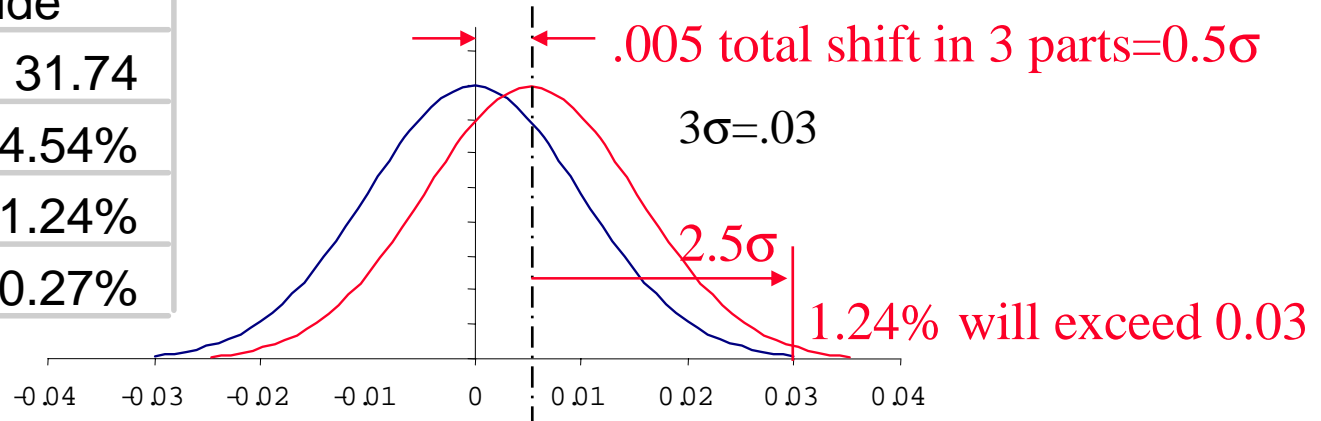
Source:

Figure 5-20 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Effect of Mean Shift



Range	% Outside
± 1 sigma	31.74
± 2 sigma	4.54%
± 2.5 sigma	1.24%
± 3 sigma	0.27%



$C_{pk} = 1.00$ $C_{pk} = .833$

How Could Non-zero Mean Variations Occur?

- For material removal processes, operators stop as soon as the part enters the tolerance zone from the high side
- For material-adding processes, operators stop as soon as the part enters the zone from the low side

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Source:

Figure 5-24 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Removal Process

Additive Process

Statistical vs Worst Case - Summary

- Statistical is cheaper because looser tolerances are allowed at each part or feature (cost rises as tolerances are tightened)
- Worst case guarantees interchangeable parts, even at batch size of one
- Statistical achieves interchangeability with some probability < 1 , enhanced if there is a big bin of parts to choose from. The likelihood of the worst combination actually being picked is pretty low and is usually easy to recover from.

Enabling Tree for SPC

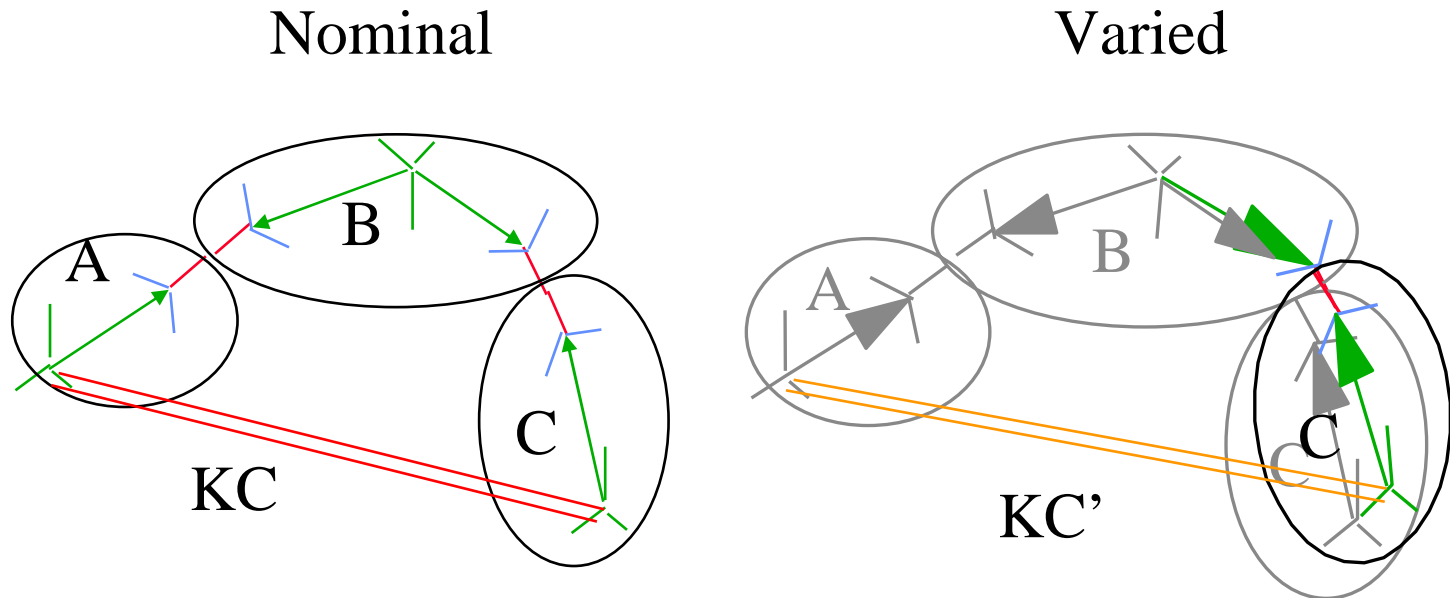
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Source:
Figure 5-23 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Math Models of Variation

- There are tolerance standards for designing single parts (ANSI Y 14.5 xx)
- There is no standard for tolerancing assemblies
- People just extend the part standard or finesse
- The following slides show how to use the 4x4 matrix method for both nominal part-part location modeling and variation modeling

Nominal and Varied KC Relationships



Each relationship, internal or external, nominal or varied, is described by a 4x4 transform

- Internal relationship
- External relationship
- = Key Characteristic

Inside a Car Engine

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Source:

Figure 5-2 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Chain of Frames for Engine

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Source:

Figure 5-3 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Valve and Lifter Use Selective Assembly

Statistical Tolerancing
cannot deliver this KC

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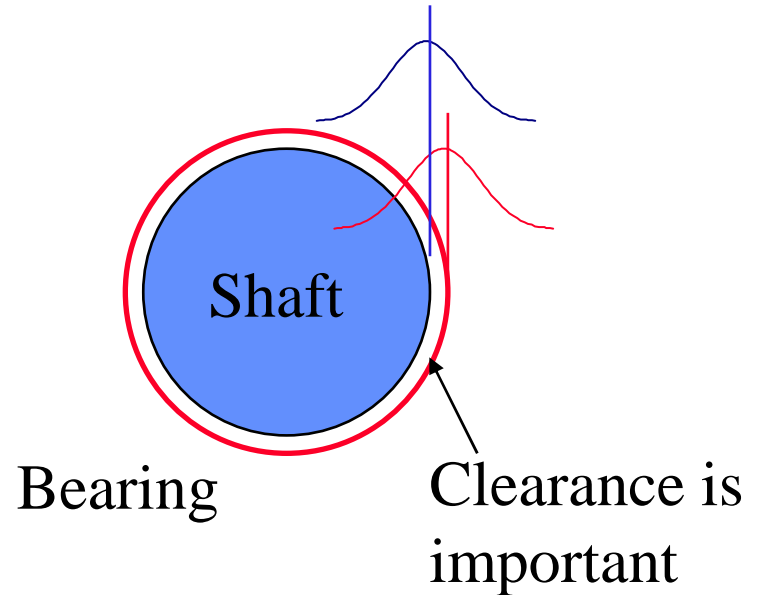
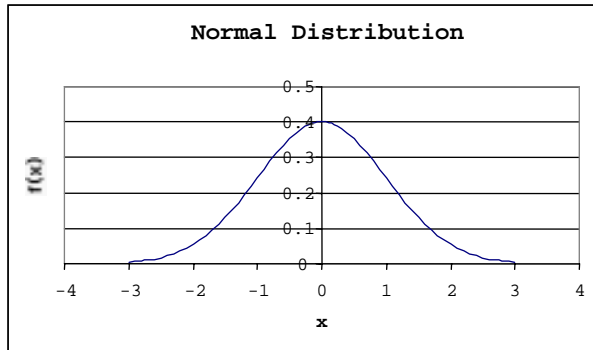
Figure 6-42 in [Whitney 2004] Whitney, D. E.
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Figure 5-5 in [Whitney 2004] Whitney, D. E.
*Mechanical Assemblies: Their Design, Manufacture,
and Role in Product Development.*
New York, NY: Oxford University Press, 2004.
ISBN: 0195157826.

How Selective Assembly is Done



Pick and measure a shaft.

If it is a bit big, pick a big bearing to get clearance right.

If it is a bit small, pick a small bearing.

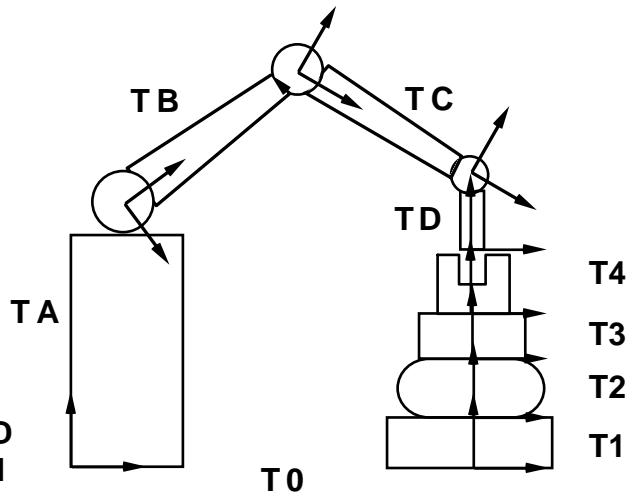
For this to work over a long stretch, there

must be about the same number of big

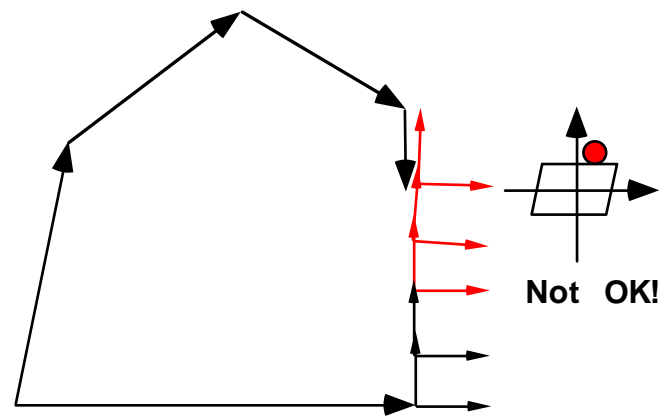
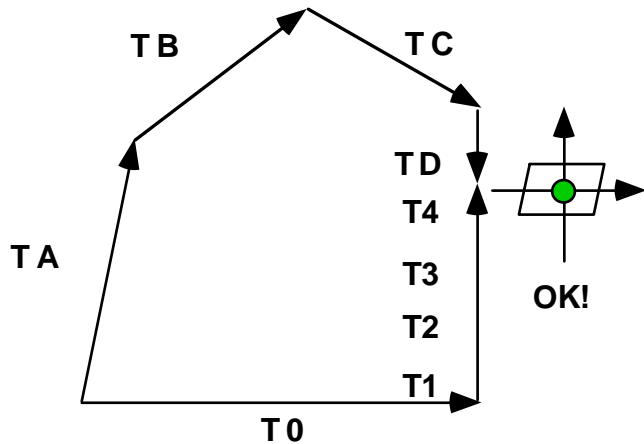
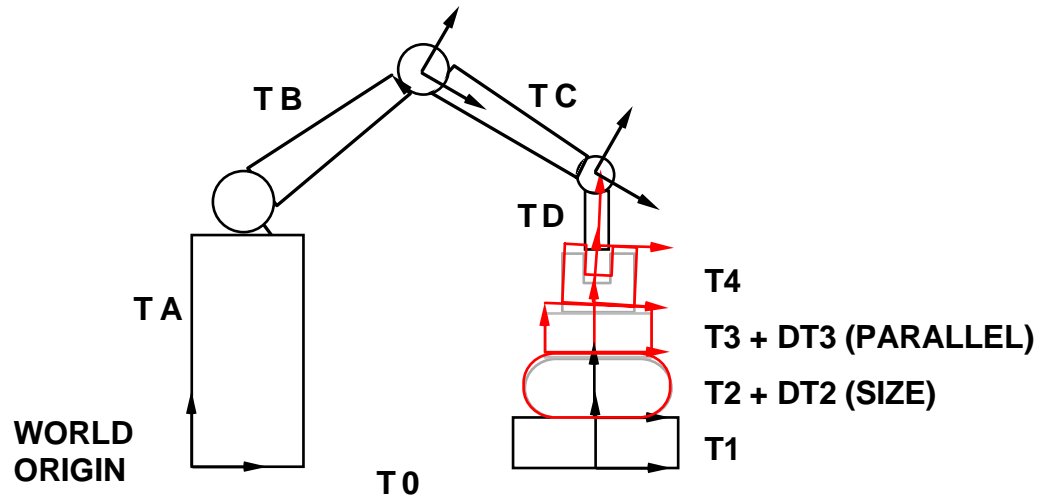
shafts as big bearings, and the same for small ones.

Assembly Process Tolerances

NOMINAL



SHOWING TOLERANCES



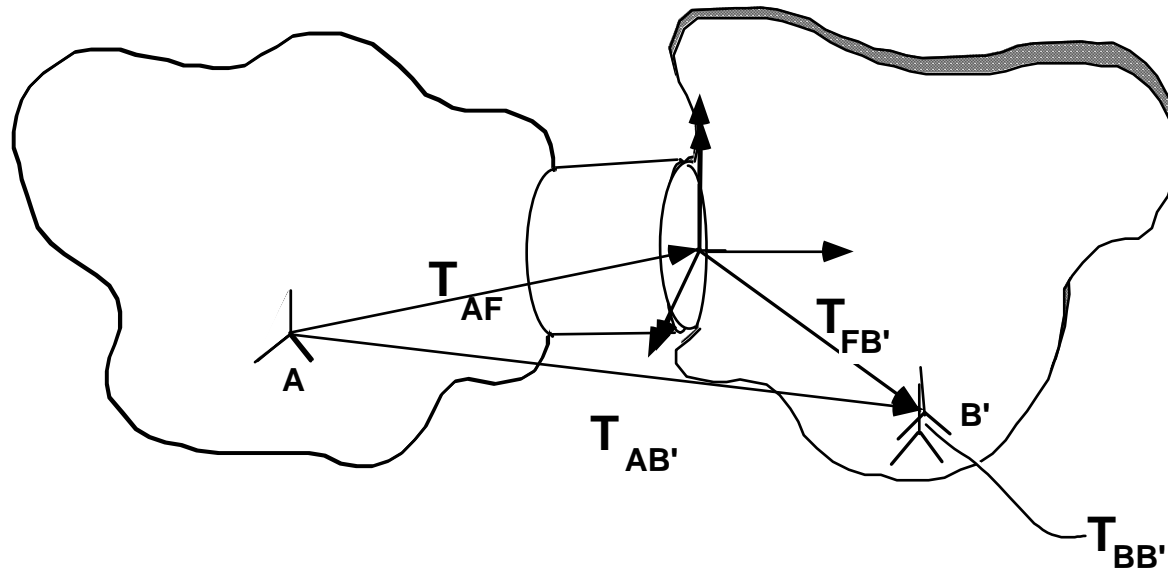
Nominal Mating of Parts

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Source:

Figure 3-17 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Varied Part Location Due to Tolerances



The varied location of Part B can be calculated from the nominal location of Part A. This process can be chained to Part C, etc., including errors on Part B. **It uses the same math as the nominal model.**

Equations for Connective Models

Nominal

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Source:

Figure 3-19 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Varied

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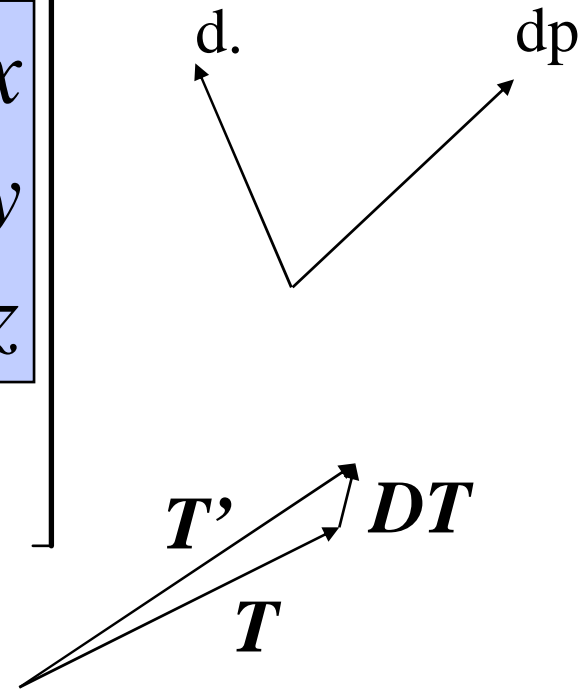
Source:

Figure 3-20 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Error Transform (Linearized)

$$DT = \begin{bmatrix} 1 & -\delta_z & \delta_y & dx \\ \delta_z & 1 & -\delta_x & dy \\ -\delta_y & \delta_x & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T' = T * DT$$



Angle Errors Are Important

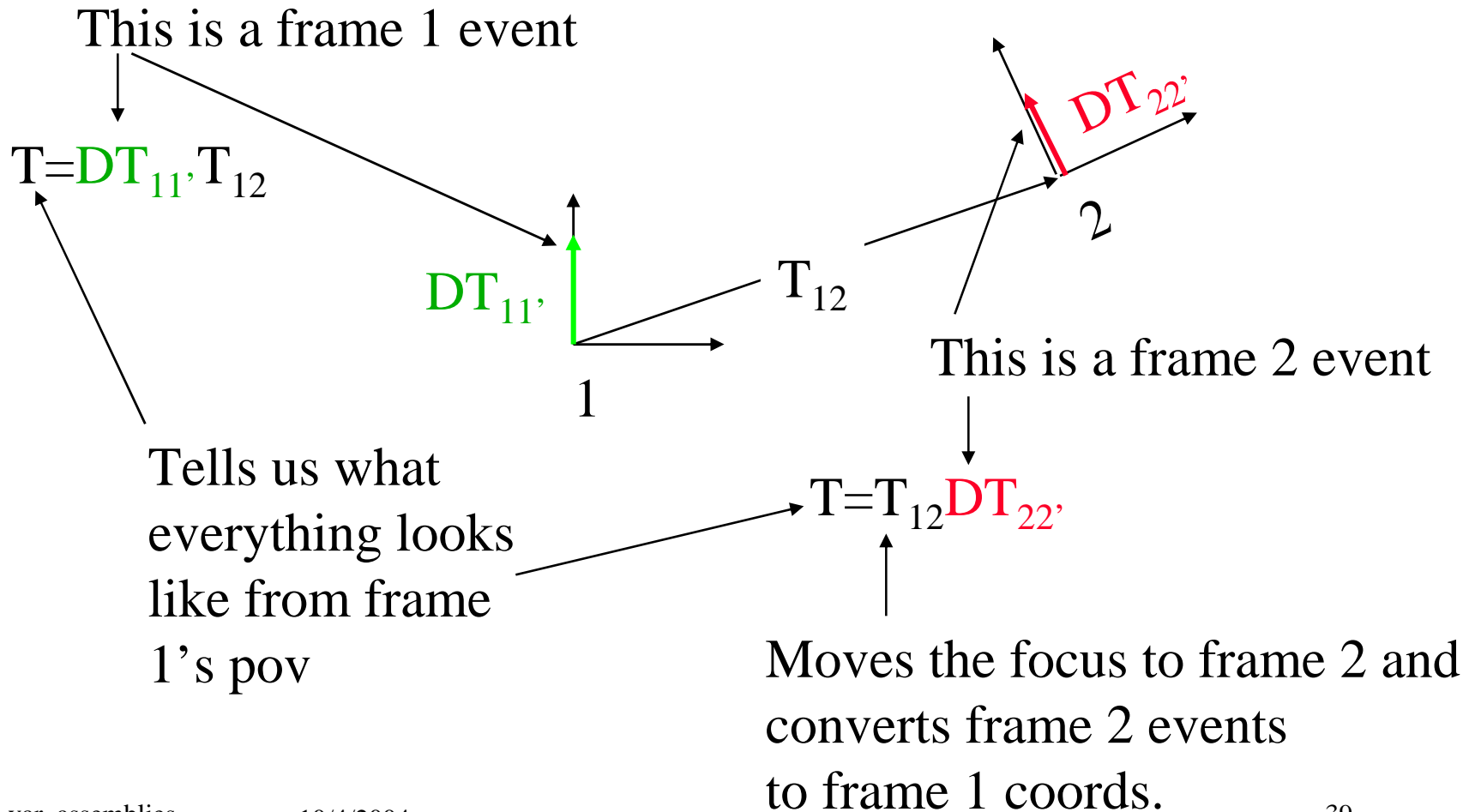
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Please see:

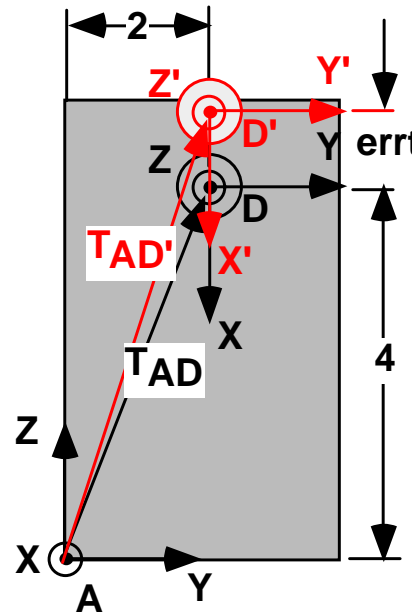
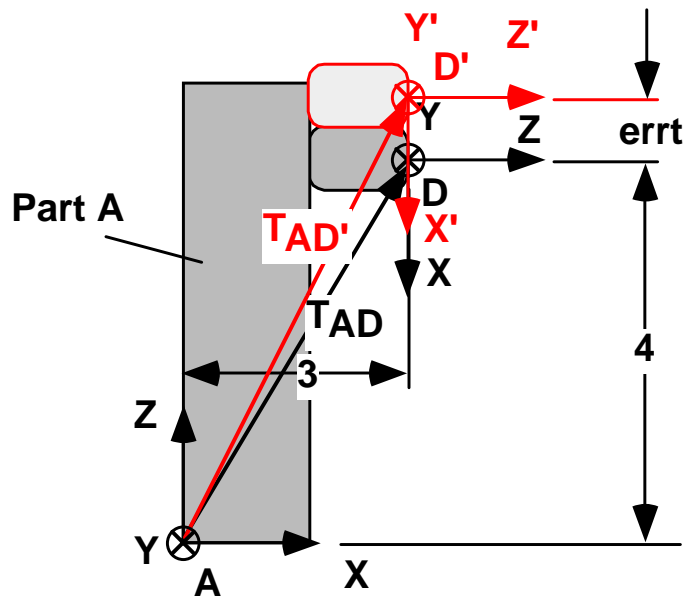
Kedrosky, Paul S. "In the Fray: What the Duffers Can Teach Tiger Woods."

The Wall Street Journal (April 14, 2004): D.16.

Transform Order is Important



Peg Position Error



```
>> T_AD = trans(3,2,4) * roty(dtr(90))
% dtr converts degrees to radians
```

$$= \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
% First method
```

```
>> DZ = errt
```

```
>> DT_AD1 = trans(0,0,DZ)
```

```
T_AD'1 = DT_AD1 * T_AD
```

```
% Second method
```

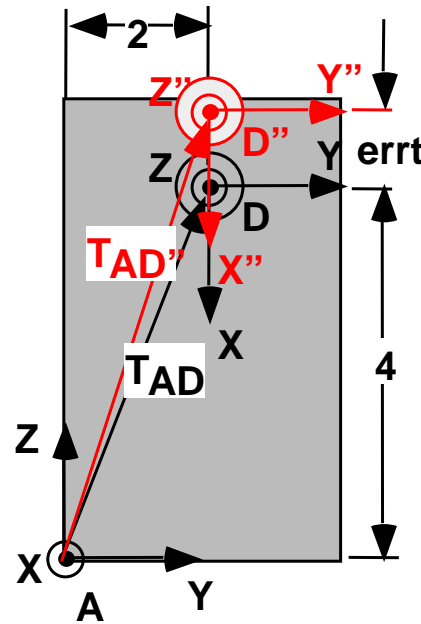
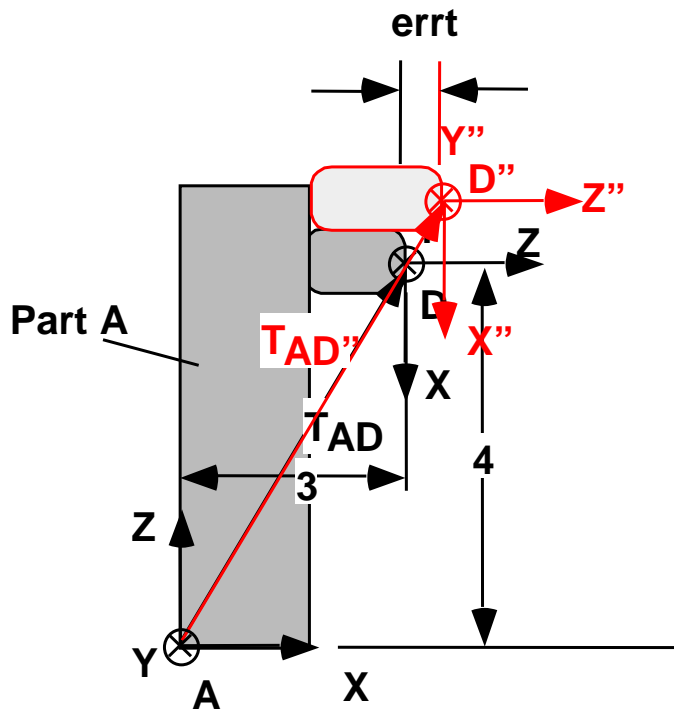
```
>> DX = errt
```

```
>> DT_AD2 = trans(-DX,0,0)
```

```
>> T_AD'2 = T_AD * DT_AD2
```

```
>> T_AD'2 = T_AD * T_DD'
```


Peg Length Error

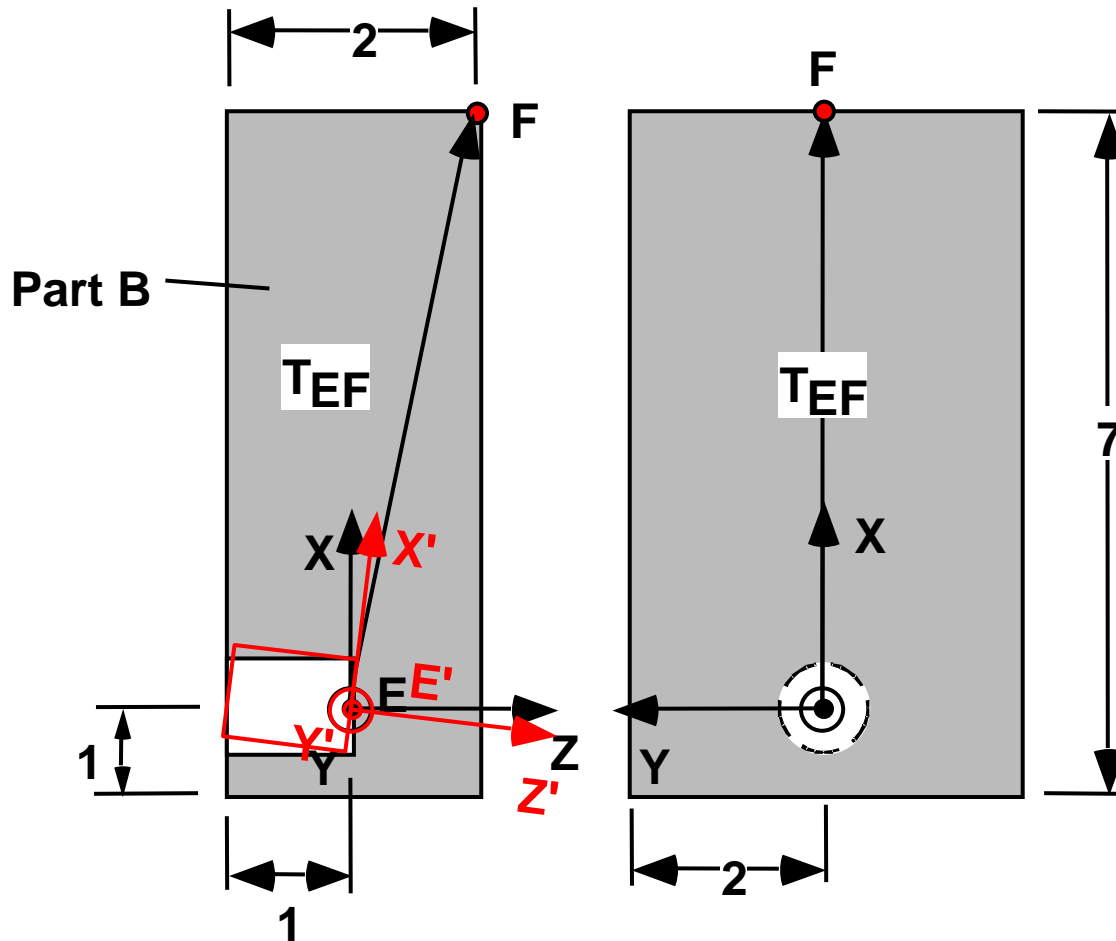


$\gg T_{AD} = \text{trans}(3,2,4) * \text{roty}(dtr(90))$
 % *dtr* converts degrees to radians

$$= \begin{Bmatrix} 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

$\gg DT_{pp} = \text{trans}(0,0,\text{randn}*.003)$

Hole Angle Error



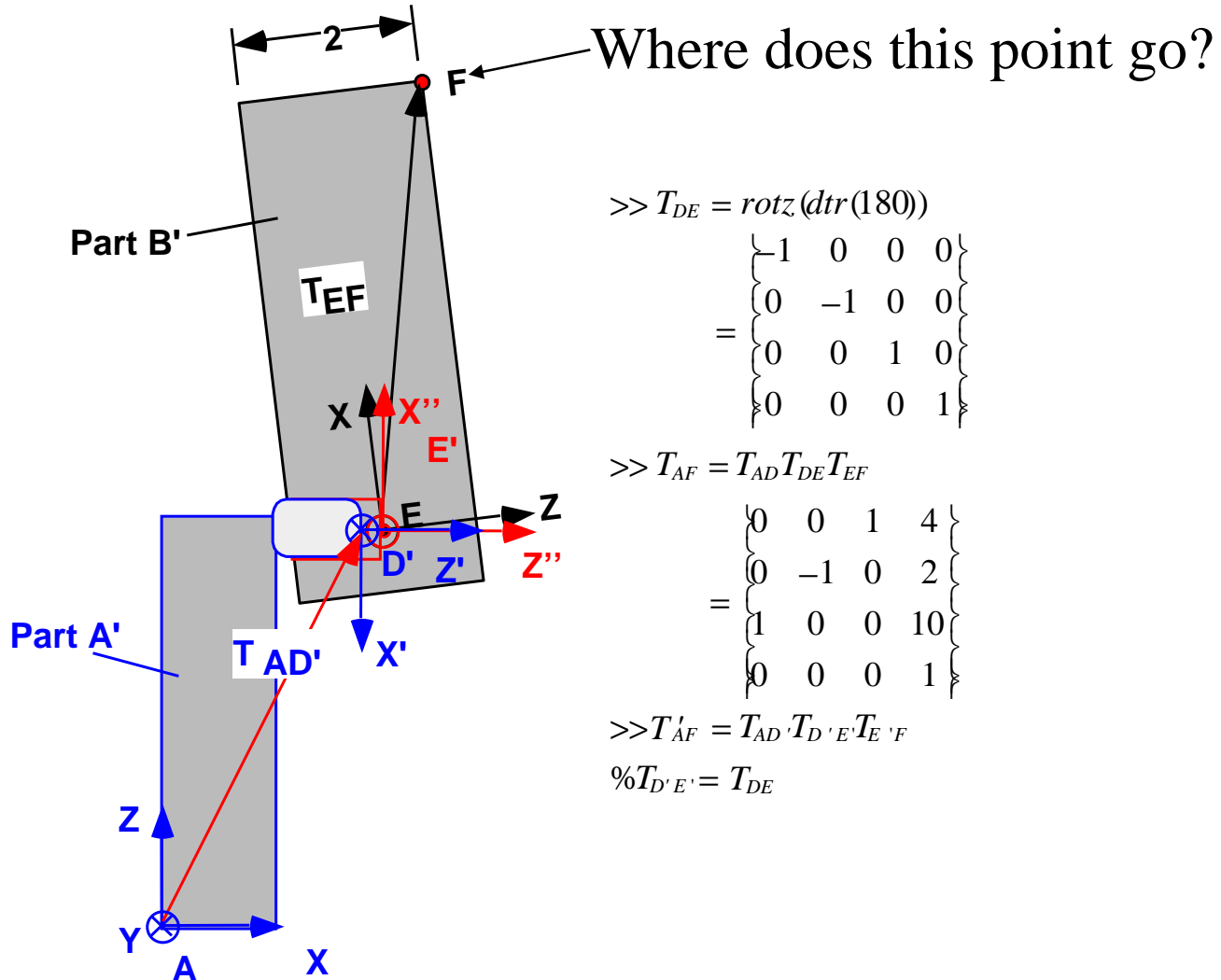
$$\gg T_{EF} = \text{trans}(6,0,1)$$

$$= \begin{Bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

$$\gg DT_{EE'} = \text{roty}(\text{dtr}(\text{erra}))$$

$$\gg T_{E'F} = DT_{E'E} T_{EF}$$

Final Variation at Assembly Level



$$\gg T_{DE} = \text{rotz}(\text{dtr}(180))$$

$$= \begin{Bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

$$\gg T_{AF} = T_{AD'} T_{DE} T_{EF}$$

$$= \begin{Bmatrix} 0 & 0 & 1 & 4 \\ 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

$$\gg T'_{AF} = T_{AD'} T_{D'E'} T_{E'F}$$

$$\%T_{D'E'} = T_{DE}$$

MATLAB for Example

```
»TAD=trans(3,2,4)*roty(dtr(90))
```

TAD =

```
0 0 1 3
0 1 0 2
-1 0 0 4
0 0 0 1
```

```
»TDE=rotz(dtr(180))
```

TDE =

```
-1 0 0 0
0 -1 0 0
0 0 1 0
0 0 0 1
```

```
»TEF=trans(6,0,1)
```

TEF =

```
1 0 0 6
0 1 0 0
0 0 1 1
0 0 0 1
```

```
»TAF=TAD*TDE*TEF
```

TAF =

```
0 0 1 4 ← TAF(1,4)
0 -1 0 2
1 0 0 -2
0 0 0 1
```

```
for i=1:10000
```

```
DTad=trans(randn*.003,0,0);
```

```
DTdpp=trans(0,0,randn*.003);
```

```
DTee=roty(dtr((randn*.003)));
```

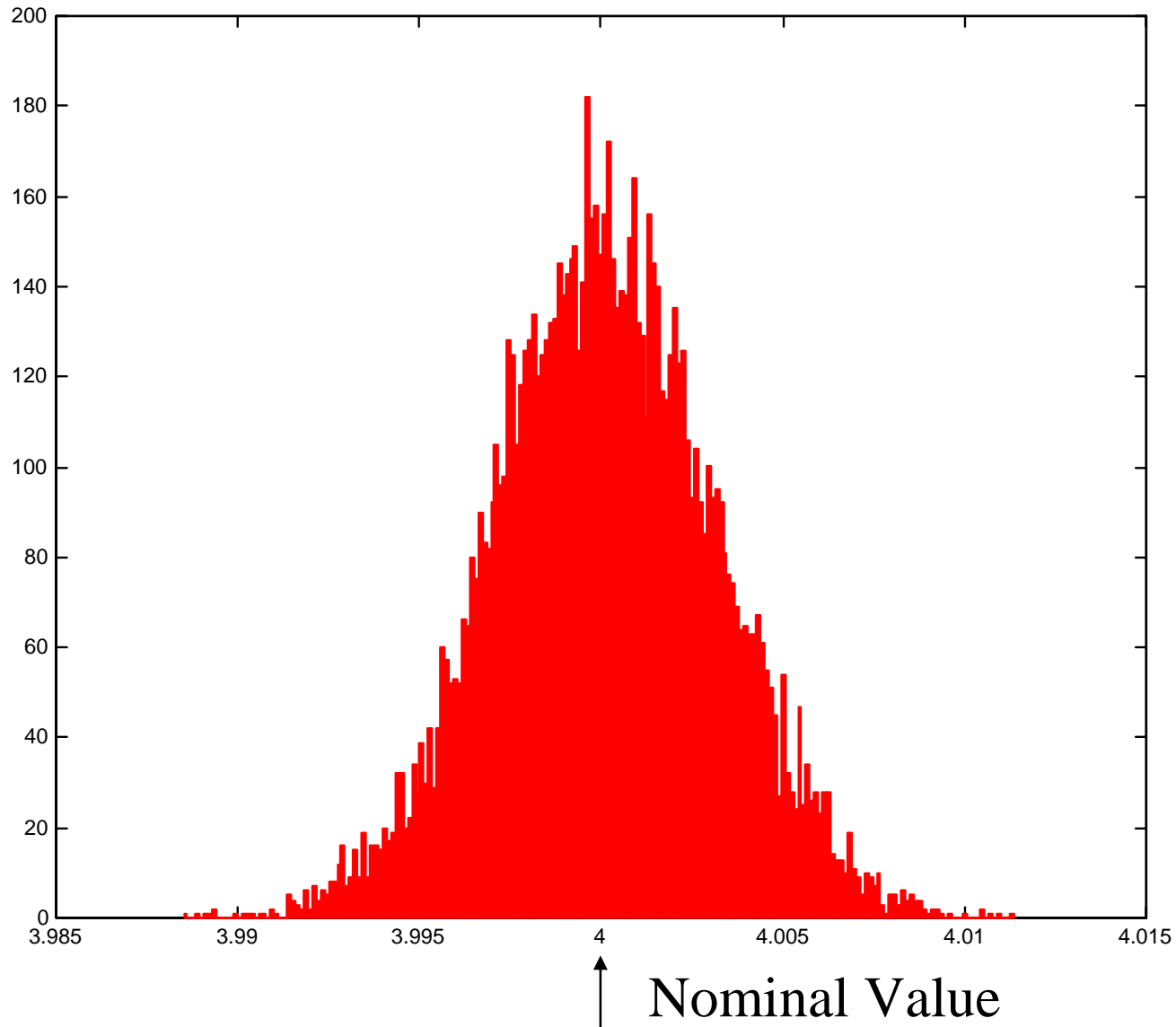
```
TAF=TAD*DTad*DTpp*TDE*DTee*TEF;
```

```
m(i)=TAF(1,4);
```

```
end
```

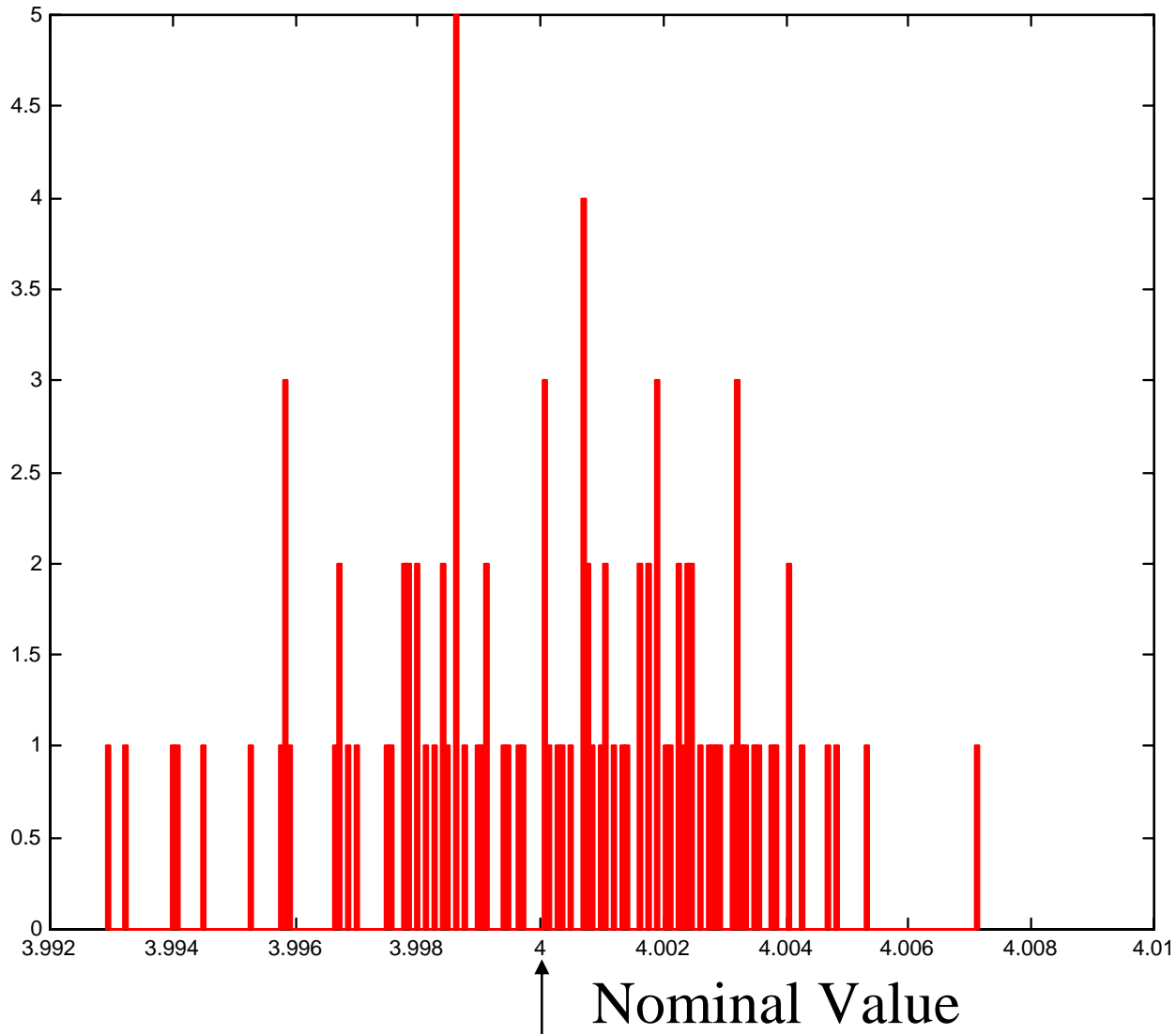
```
»hist(m,200)
```

Result for 10000 Samples



va

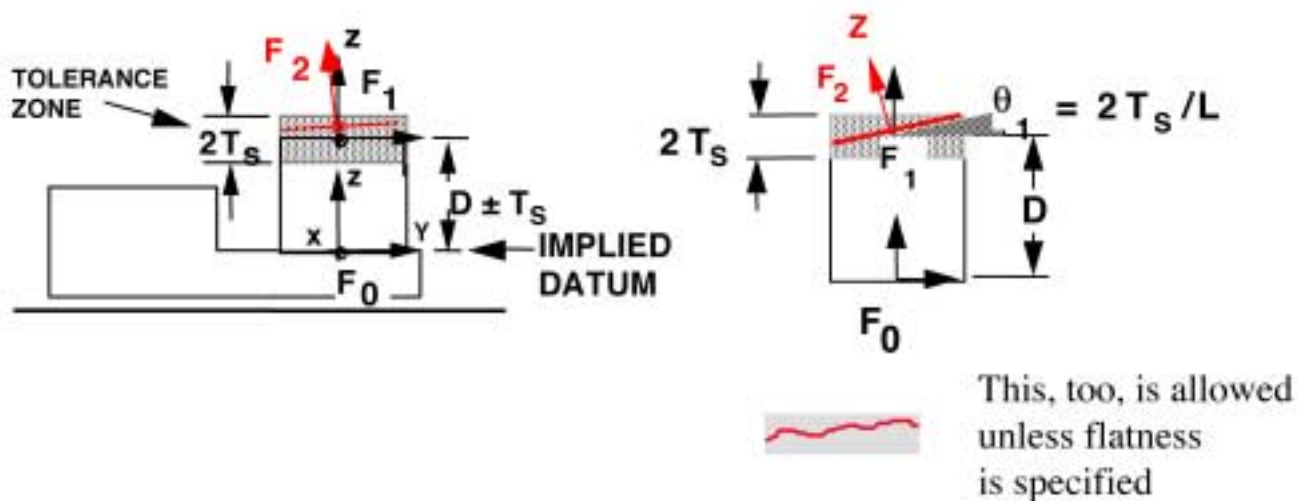
Histogram of 100 Samples



Constructing Matrix Representations of GD&T

The Actual Surface Must Lie Inside the Zone

Rule #1: The surface must be correct at max material condition.



Position and Angle are Coupled by Rule #1

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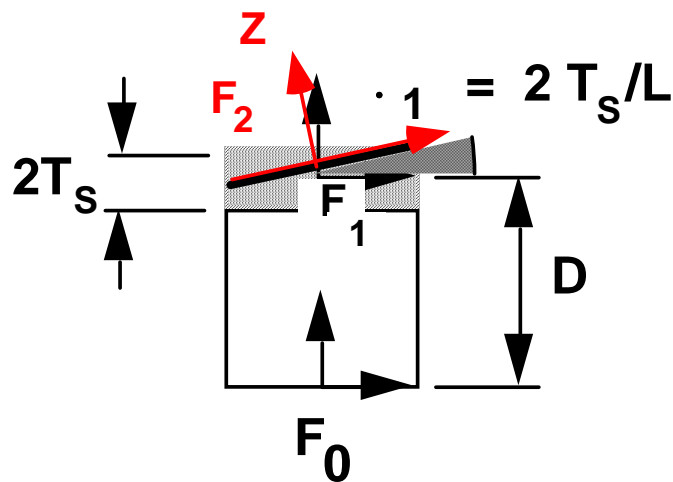
Source:

Figure 6-15 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

f_{opt} is derived by simulation to give the best Gaussian fit to the actual diamond

Tolerance Zones as 4x4 Transforms

Two dimensional example:



$$T_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & D \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{1,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$DT_{1,2} : \begin{aligned} dX &= 0 \\ dY &= 0 \\ dZ &= f_{opt} \times T_s \end{aligned}$$

$$\begin{aligned} \delta_x &= f_{opt} \times 2T_s / L_y \\ \delta_y &= f_{opt} \times 2T_s / L_x \\ \delta_z &= 0 \end{aligned}$$

$$f_{opt} = 0.95$$

$$T_{0,2} = T_{0,1} * T_{1,2} * DT_{1,2}$$

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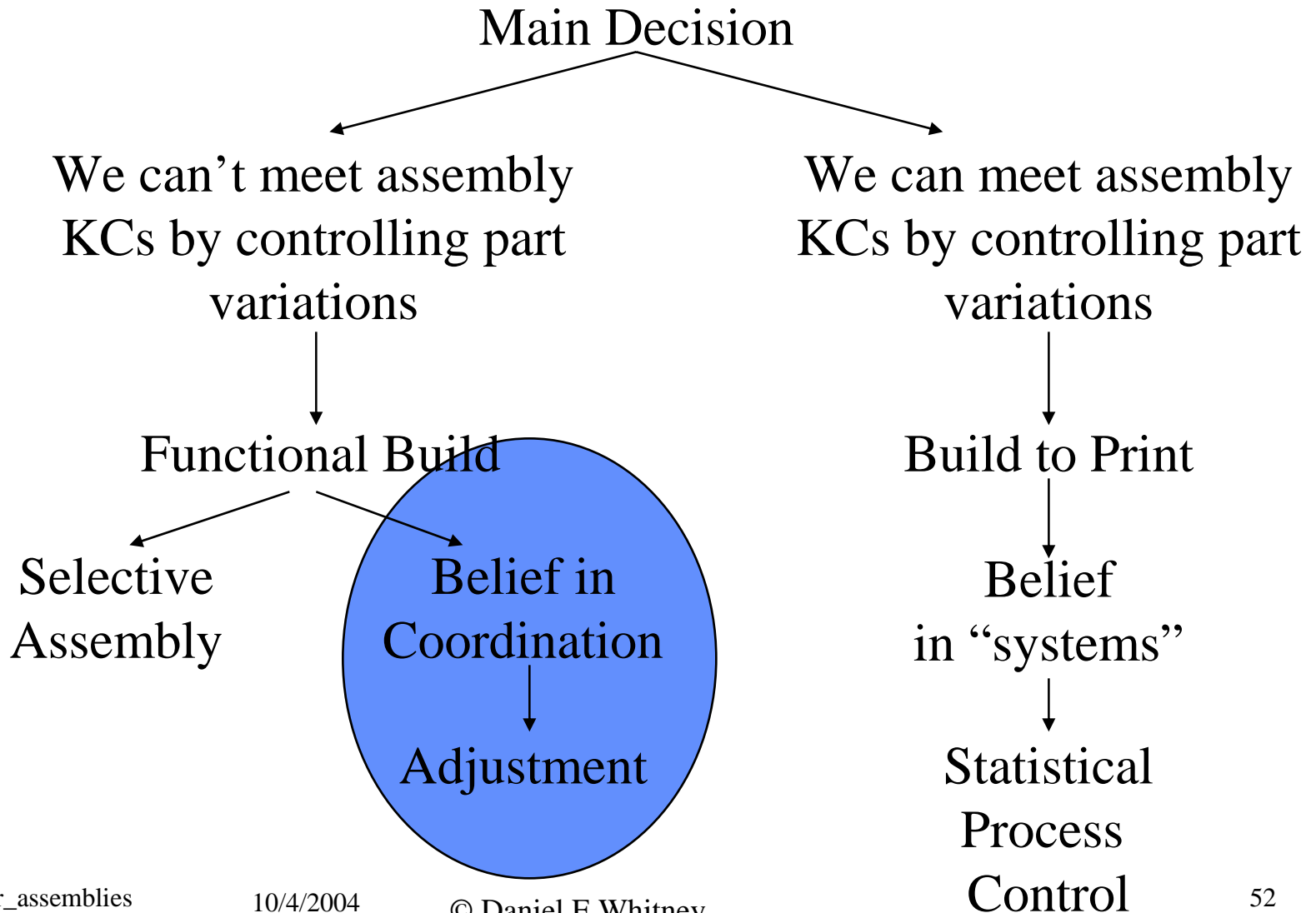
Source:

Table 6-1 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Two Ways to Use This Model

- Direct Monte Carlo simulation (can use Matlab)
 - randomly select dX , dY , δ_x , etc (eliminate parts that violate Rule #1) (use $f_{\text{opt}} = 1$) Assume that $T_s = 3\sigma$
$$dX = \text{randn} * T_s / 3; \delta_x = \text{randn} * 2 * T_s / (3 * LY)$$
 - multiply out all the Transforms
 - repeat many times, collect histograms
- Plug into a closed form solution (use f_{opt} to approximate Rule #1)
 - requires assuming a Gaussian distribution for each of the errors
 - delivers ellipsoids of location and angle error at the end of the transform chain

Logic Tree of Control of Variation



Functional Build

- Accept the fact that the mean cannot be brought to the desired value:
 - Too much trouble, time, or cost
 - Too much variation in the mean
 - Some other mean may be shifted the other way
- Coordinate the parts involved
- Keep an eye on things and make adjustments
- This is often done on tools and dies if there is one source (one die) for each part involved
- Then the focus is to control variation around the mean
- The SPC metric is then C_p

$$C_p = \left[\frac{USL - LSL}{6\sigma} \right]$$

C_p Ignores Mean Shift

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Source:

Figure 5-19 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.

Logic Tree for Tolerancing

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Source:

Figure 6-48 in [Whitney 2004] Whitney, D. E. *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development*. New York, NY: Oxford University Press, 2004. ISBN: 0195157826.